EXERCISE DUE MONDAY, 10/3

(1) The probability space is \( S = \{1, 2, 3, 4, 5\} \). The probability of every point \( \omega \) in \( S \) is \( 1/5 \). We define random variables \( X_i \) by

\[
\begin{align*}
X_1(5) &= 2 \\
X_1(\omega) &= 1 & \omega < 5 \\
X_2(1) &= 2 \\
X_2(\omega) &= 1 & \omega > 1 \\
X_3(3) &= 2 \\
X_3(\omega) &= 1 & \omega \neq 3.
\end{align*}
\]

Let \( \mathcal{F}_t, t = 1, \ldots, 3 \) be defined as the \( \sigma \)-field generated by \( X_s, s < t \).

(a) Display the sets making up each of \( \mathcal{F}_1, \mathcal{F}_2 \) and \( \mathcal{F}_3 \).
(b) Could these three random variables form part of a stationary process?
(c) Find \( \text{Cov}(X_i, X_j) \) for all combinations of \( i, j = 1, \ldots, 3 \).
(d) Find \( E_t[X_3] \) and \( \text{Var}_t[X_3] \) for \( t = 1, 2 \), evaluated at \( X_1 = 1, X_2 = 1, \) at \( X_1 = 2, X_2 = 2, \) and at \( X_1 = 2, X_2 = 1 \). Note that these variables are not joint normal; the conditional expectations will not be linear functions.

(2) (a) For each of the sets of moving average weights \( a \) below, compute and plot the acf of \( X_t = \sum a_i \varepsilon_{t-i} \) for time separations \( s = -15, \ldots, 15 \). This will be tedious unless you use the computer.

(b) For each of the sets of moving average weights \( a \) below, compute and plot 5 simulated draws for \( X_t, t = 1, \ldots, 50 \) by generating 60 i.i.d. \( N(0,1) \) random draws and averaging them with \( a \). Note that you can draw a single set of 5 i.i.d. \( \varepsilon \) sequences and use the same 5 for each of the \( a \)'s. This makes it clearer what differences are due to the \( a \)'s alone. All 5 lines for a single \( a \) should be on the same plot.

The \( a \)'s:

(a) \( a_i = 1, i = 0, \ldots, 10 \)
(b) \( a_i = \sin(2\pi i/10) + 1, i = 0, \ldots, 10 \)
(c) \( a_i = \cos(2\pi i/10) + 1, i = 0, \ldots, 10 \)
(d) \( a_i = (-1)^i, i = 0, \ldots, 10 \)

Command to generate a 60 by 5 matrix of \( N(0,1) \) random variables:

matlab: \( z = \text{nrnd}(60,5) \)
R: \( z \leftarrow \text{matrix(rnorm(60*5),ncol=5)} \)

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(3) There are, or will be, monthly data on the Federal Funds rate on the course web site. Using these data, find a maximum likelihood estimate of the weights $a$ in a 12th-order Gaussian MA model with constant mean $\bar{r}$ for these data. Determine whether the MA weights to which your estimates have converged are fundamental. [You can use “root-flipping”, which we will probably cover in the 9/28 lecture, or you can try starting the maximization from a different place to get convergence to a different set of weights, so you can compare $a_0$’s, or you can construct an approximation to the one-step-ahead predictor by using a large finite number of lags and see if its residual variance is close to $a^2_0$.]