NOTES ON THE `GENSYS2` PACKAGE

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This package assumes the user has a model in the form

$$K(w_t, w_{t-1}, \varepsilon_t) + \Pi\eta_t = c\ , \tag{1}$$

where $E_t\eta_{t+1} = 0$ and $E_t\varepsilon_{t+1} = 0$. The disturbances ε_t are exogenously given, while η_t is determined as a function of ε when the model is solved. There are n equations and n elements in w.

We assume the model has a deterministic steady state $\bar{w}$ satisfying

$$K(\bar{w}, \bar{w}, 0) = C\ . \tag{2}$$

We assume that the solution will imply that w_t remains always on a stable manifold, defined by $H(w_t) = 0$ and satisfying

$$\{H(w_t) = \gamma \quad \text{and} \quad K(w_{t+1}, w_t, \varepsilon_{t+1}) + \eta_{t+1} = C\} \;\Rightarrow\; H(w_{t+1}) = \gamma\ , \tag{3}$$

when η_{t+1} depends on ε_{t+1} and w_t appropriately. The "stability" of this manifold means either that $w_t \to \bar{w}$ along it, or that in the neighborhood of $\bar{w}$ w_t diverges from $\bar{w}$ at a rate no faster than some known bound. It is this condition that allows us to solve for η.

The system (1) has the second-order Taylor expansion about $\bar{w}$

$$\begin{aligned} K_{1ij}dw_{jt} = K_{2ij}dw_{j,t-1} + K_{3ij}\varepsilon_{jt} + \\ \tfrac{1}{2}(K_{11ijk}dw_{jt}dw_{kt} + 2K_{12ijk}dw_{jt}dw_{k,t-1} + 2K_{13ijk}dw_{jt}\varepsilon_{kt} \\ + K_{22ijk}dw_{j,t-1}dw_{k,t-1} + 2K_{23ijk}dw_{j,t-1}\varepsilon_{kt} + K_{33ijk}\varepsilon_{jt}\varepsilon_{kt})\ , \end{aligned} \tag{4}$$

where we have resorted to tensor notation. That is, we are using the notation that

$$A_{ijk}B_{mnjq} = C_{ikmnq} \quad \Leftrightarrow \quad c_{ikmnq} = \sum_j a_{ijk}b_{mnjq}\ . \tag{5}$$

where a, b, c in this expression refer to individual elements of multidimensional arrays, while A, B, C refer to the arrays themselves. As special case, for example, ordinary matrix multiplication is $AB = A_{ij}B_{jk}$ and the usual matrix expression $A'BA$ becomes $A_{ji}B_{jk}A_{km}$. Note that we are distinguishing the array K_{mij} of first derivatives from the array K_{mnijk} of second derivatives only by the number of indexing subscripts the two arrays have.

The solution will take the form

$$y_t = F(y_{t-1}, \varepsilon_t) \tag{6}$$

$$x_t = M(y_t)\ , \tag{7}$$

where $[y_t'\ x_t']' = Z'w_t$, with Z a square, non-singular matrix. Equation (7) is just (3) solved for some particular linear combination of w's (i.e. x). If the original system has a standard "state-control" form, y can be taken as the state variable and x as the control variable. However the `gensys2` program does not require that the system be put in this form to start with. If it will aid interpretation to specify the state vector in advance, the program will use the prespecified y vector; but it can also generate a y on its own.

The second-order expansion of the solution is

$$\begin{aligned} dy_{it} =& F_{1ij} dy_{j,t-1} + F_{2ij}\varepsilon_{jt} + F_{3i}\sigma^2 \\ &+ \tfrac{1}{2}(F_{11ijk} dy_{j,t-1} dy_{k,t-1} + 2F_{12ijk} dy_{j,t-1}\varepsilon_{kt} + F_{22ijk}\varepsilon_{jt}\varepsilon_{kt}) \end{aligned} \tag{8}$$

$$dx_{it} = M_{11ijk} dy_{jt} dy_{kt} + M_{2i}\sigma^2\ . \tag{9}$$

The program's calling sequence is

$$[\mathtt{FD}, \mathtt{FDD}, \mathtt{M11}, \mathtt{M2}, \mathtt{C}, \mathtt{q}, \mathtt{zs}, \mathtt{zu}, \mathtt{v}, \mathtt{gev}, \mathtt{eu}] \\ = \mathtt{gensys2}(\mathtt{KD}, \mathtt{KDD}, \mathtt{c}, \mathtt{Pi}, \mathtt{omega}, \mathtt{pick}, \mathtt{div}).$$

KD: A 1×3 cell array with KD{j}= $K_{j\cdot\cdot}$ from (4).

KDD: A 3×3 cell array with KDD{jkm}= $K_{jk\cdots}$ from (4).

c: The right-hand side of (1).

Pi: The coefficient on η from (1).

omega: The normalized covariance matrix of ε, i.e. Ω from $\mathrm{Var}(\varepsilon(t)) = \sigma^2\Omega$.

pick: Candidate weights to form y. This is a matrix Ψ such that $y_t = \Psi w_t$. This argument is optional. If it is missing, or implies an incorrect selection of the state, the program will supply its own Ψ.

div: Dividing line between unstable and "stable" roots. This argument is optional. If it is missing, absolute values of 1 or less are assumed stable and anything larger than 1 is assumed unstable.

FD: A 1×3 cell array with FD{j}= $F_{j\cdot\cdot}$ from (8).

FDD: A 2×2 cell array with FDD{j,k}= $F_{jk\cdots}$ from (8).

C: The vector $[\bar{y} - F_1\bar{y}\ ;\ \bar{x}]$.

q,v: The matrices returned as `q,v` from `[a,b,q,z,v]=qz(KD{1},KD{2})`, after sorting to put unstable generalized eigenvalues at the bottom .

zs,zu: `[zs;zu]'` is the matrix returned as `z` by the `qz` call above. If `pick` was not present or was overridden, $y_t =$ `zs` w_t. Always $x_t =$ `zu` w_t .

gev: The generalized eigenvalues of $K1, K2$, sorted.

eu: Existence and uniqueness indicator. See the program's help notes for details.