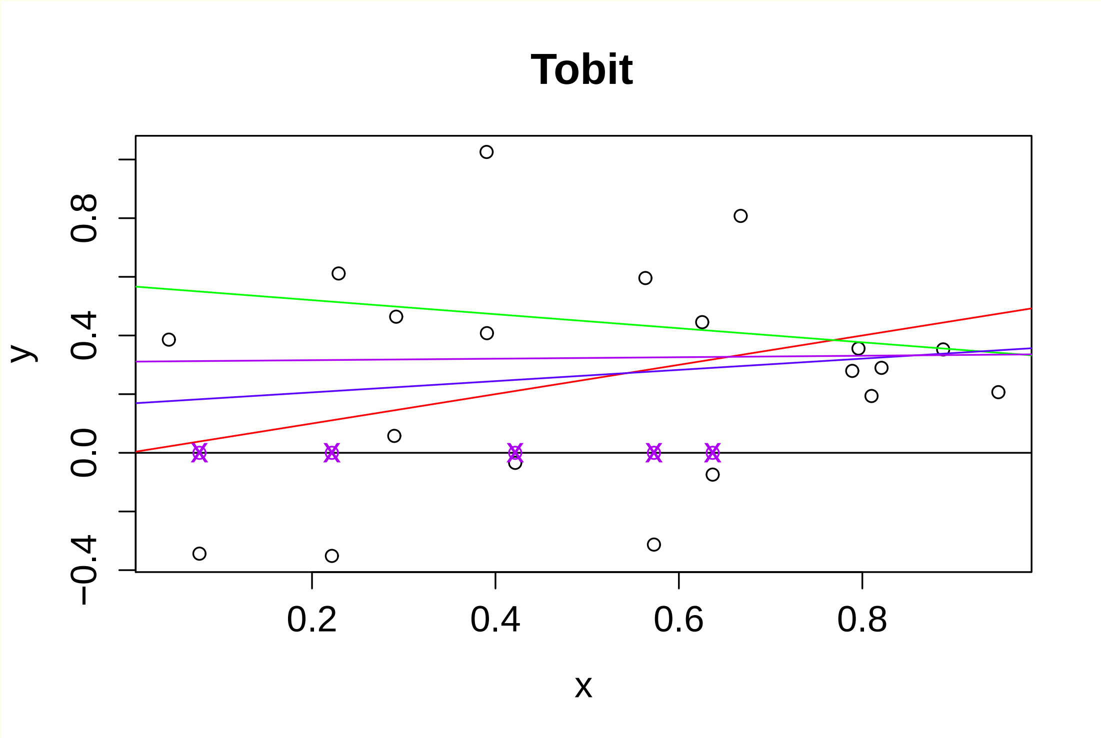
Limited dependent variables

January 16, 2014

Tobit

Why Tobin thought it up. Truncation vs. censoring.



Tobit

$$y_t = \max 0, X_t\beta + \varepsilon_t \;, \qquad \varepsilon_t \mid X_t \sim N(0, \sigma^2)$$

pdf for y_t is over lump of size 1 on 0 and Lebesgue measure on $(0, \infty)$. Thus it is

$$p(y_t \mid \beta) = \begin{cases} \frac{1}{\sigma}\phi\left(\frac{y_t - X_t\beta}{\sigma}\right) & \text{if } y_t > 0 \\ \Phi(-X_t\beta/\sigma) & \text{if } y_t = 0 \,. \end{cases}$$

Just maximize and/or plot the likelihood. (Could do data augmentation, generating unobserved $y_t < 0$ values in a Gibbs step.)

Censoring

$$y_t \mid x_t \sim \phi\left(\frac{y_t - x_t\beta)}{\sigma}\right)\left(1 - \Phi\Big(\frac{y_t - x_t\beta)}{\sigma}\Big)\right)^{-1}$$

That is, $y_| x_t$ is $N(x_t\beta, \sigma^2)$ *truncated* below at 0. The lower tail of the normal is missing, so to get the correct conditional distribution we have to scale up the normal pdf so it integrates to one over the truncated range.

For both cases

- Unlike SNLM, here the normality assumption matters, even for large samples. So we could also do Tobit-t, or consider even fancier (asymmetric, e.g.) pdf's for $\varepsilon_t \mid X_t$.

- The lower tail is identified asymptotically if $X_t\beta$ gets arbitrarily large, often. But even when it's not ID'd, tracing out how a prior on the distribution shape affects results is worthwhile.

LPM

Logit, Probit

Measures of fit for LDV models