EXERCISE ON OLS, PROBABILITY INTERVALS, ASYMPTOTICS

(1) For each of the \{X_n\}, Z pairs below, determine whether \(X_n \overset{Q}{\to} Z\), where Q takes on the values \(P, D, q.m., \text{ and a.s.}\). Explain your answers. Note that in some of the cases where there is convergence in probability, determining whether there is a.s. convergence may be challenging.

You can use the central limit theorem and the strong law of large numbers.

(a) \(X_n = Z + Y_n\), where \(Y_n\) is \(N(0, 1/n)\) for each \(n\) and independent across \(n\).

(b) \(X_n\) independent across \(n\), \(P[\{X_n = 1/n\} = 1 - 1/n, P[\{X_n = \sqrt{n}\} = 1/n, P[\{Z = 0\} = 1.\]

(c) \(X_n = \sum W_i/n + 1/n, \{W_i\}\) is an i.i.d. sequence with finite variance and \(E[\{W_i\} = \mu, P[\{Z = 0\} = 1.

(d) \(X_n = \sum W_i/\sqrt{n + \sqrt{n}}, \{W_i\}\) i.i.d. with mean zero and variance 1, \(Z \sim N(0, 1).

(e) \(X_n = \sum W_i/\sqrt{n + \sqrt{n}}, \{W_i\}\) i.i.d. with mean zero and variance 1, \(Z \sim N(0, 1).

(2) Compute a linear regression of testscr on str, avginc, meal_pct el_pct teachers and computer, using the caschool data set.

(a) Form a \(\chi^2\)-squared or \(F\) statistic to test the hypothesis that the coefficients on teachers and computer are both zero. Comment on whether it suggests these coefficients as a pair are unimportant in explaining testscr. [In R, the easiest way to do this is to get the package car (companion to applied regression), which you should be able to get with install.packages(car). You then load the package with library(car). After that several useful commands are available, including linearHypothesis(), which provides an easy way to generate test statistics like the one you are asked for here. With slightly more effort, you can do without car and use the function vcov(lmout) (to get the coefficient covariance matrix), then a matrix expression to get the \(\chi^2\)-squared or \(F\) statistic, then pchisq() or pf().]

(b) Find the estimated correlation matrix of the coefficients on these two variables, and use it to sketch a few representative level curves of their flat-prior joint posterior pdf in \(\mathbb{R}^2\). [In R, vcov(lmout) delivers the coefficient covariance matrix for a linear regression, and cov2cor(W) converts the covariance matrix \(\hat{W}\) to a correlation matrix. The car package has a function that computes and plots confidence ellipsoids (which are also flat-prior probability ellipsoids) from the output of lm.]

(c) Both teachers and computer are simple counts over the whole school, so they vary with the size of the school. If the size of the school were important, one might expect that the two variables would enter with the same sign. Another possibility is that it is computers per teacher that matters, so that we would expect the two coefficients to have opposite signs. Generate a sample of 10000 draws from the joint posterior distribution on \(\beta, \sigma^2\) (the regression coefficients and the residual variance) under a flat prior on \(\log \sigma^2\) and \(\beta\). Use it to assess the probability that the coefficients on teachers and computer are:

(i) both positive,

(ii) both negative,

(iii) positive on teachers and negative on computer

(iv) positive on computer and negative on teachers.

What do these results imply about the evidence on whether there are school size effects?