ELECTION EXERCISE, CONTINUED, AND SOME ESTIMATORS

(1) In addition to parts (c) and (d) of the previous exercise, consider the problem of predicting not just the percentage of the two-party vote obtained by Bush, but the election outcome. As was illustrated in 2000, the US election system does not guarantee that the winner of the popular vote wins the election. Suppose that the probability that Bush wins is zero if he gets less than 40% of the vote, one if he gets more than 60% of the vote, and for vote percentages p between .4 and .6 the probability of winning is given by the polynomial $6q^5 - 15q^4 + 10q^3$, where $q = (p - .4)/.2$. This polynomial increases smoothly from zero to one over (.4,.6). Assuming this model for the probability of winning conditional on the percentage of vote won is correct, combine your posterior on the percentage of the vote won with this model to come up with a posterior probability that Bush wins re-election. You will probably want to solve this with simulation methods, as the integrals involved in an analytic computation are unpleasant. Carry out the computation both for your own chosen prior and for a prior flat in β and $\log \sigma^2$ (equivalent to a flat prior on $\log \sigma$ or $\log \nu = 1/\sigma^2$).

(2) We are estimating capacity output for a plant. We assume capacity $\bar{Y}$ is constant, and we have T i.i.d. observations on actual output Y_t. We believe the model

$$\log Y_t = \log \bar{Y} - \varepsilon_t \,,$$

where $\{\varepsilon_t \mid \bar{Y}\}$ has the exponential distribution (Gamma(1,1)) at each t. Note that this means, because the exponential distribution puts probability zero on negative ε_t, that the observed Y_t is always smaller than the capacity.

(a) Find an unbiased estimator of $\log \bar{Y}$.
(b) Find an unbiased estimator of $\bar{Y}$.
(c) Find a sufficient statistic for $\bar{Y}$.
(d) Find the maximum likelihood estimator for $\bar{Y}$.
(e) Find the posterior mean and median for $\bar{Y}$, assuming a flat prior on $\bar{Y}$.
(f) Find the posterior median and mean for $\bar{Y}$, assuming a prior pdf equal to $1/\bar{Y}^2$ on $(1, \infty)$ and zero for $\bar{Y} < 1$.
(g) Explain why it is likely your unbiased estimator of $\bar{Y}$ from part 2b or the estimator $\hat{\bar{Y}} = e^{\hat{\bar{y}}}$, where $\hat{\bar{y}}$ is the unbiased estimator of $\log \bar{Y}$ from part 2a, produces an inadmissible estimator. [This is a one-line argument, once you see it.]