Outline and Reading List

The main text for this part of the course (up to the October break) is DeGroot and Schervish (2002) (D&S). D&S cover most topics we will discuss, but the book is at a somewhat more elementary level than this course. D&S usually give separate treatment to discrete-probability and continuous-probability versions of the same concepts. They also usually do univariate versions first, then multivariate versions. We have to move faster than that, and will discuss the theory for the general case of distributions in $\mathbb{R}^n$ from the start. So when a topic has gone by too quickly, or without enough examples, in lecture, you may well find that reading the corresponding parts of D&S makes it clearer. Students who already have strong math background might find Schervish (1995) a better investment than D&S. It covers much more than we will cover in this course, but will be a useful reference if you go further in econometrics or finance. Gelman, Carlin, Stern, and Rubin (1995) is also a useful reference for modern Bayesian methods, a few of which we will discuss, and should be available at the PU store.

1. Probability

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1.1. **Probabilities as tangent planes in optimal decision making.**

D&S 1.1-1.5, 1.11

Ferguson (1967, chapters 1 and 2)

Both of these references include considerable material that we did not cover in class and that you are not responsible for. Ferguson gives a more rigorous and detailed discussion that covers everything we did in class. The D&S chapters discuss other approaches to interpreting probability and cover basics (like set theory) that we are assuming everyone knows to start with.

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1.2. **Distributions and densities.**

D&S chapters 2 and 3. You can skip 2.4 and 2.5 for now.

D&S maintain a distinction between distributions of random variables and random vectors, which are always in $\mathbb{R}^n$, and probability functions on the sample space (which they call S and we call Ω). They don't require that Ω be a subset of $\mathbb{R}^n$, but don't consider any frameworks that are actually more general than what we will handle. We will assume always that Ω is a subset of $\mathbb{R}^n$, or can be mapped in to such a subset. (E.g. "rain" and "clear" can be labeled "1" and "2".)

1.2.1. *Probabilities from ordinary densities in* $\mathbb{R}^n$.

1.2.2. *Discrete probabilities as densities with respect to counting measure.*

1.2.3. *Densities with respect to more complicated measures in* $\mathbb{R}^n$.

Absolute continuity. Continuity of one measure w.r.t. another.

1.2.4. *Distribution functions in* $\mathbb{R}^n$.

A way to characterize all the probability measures in $\mathbb{R}^n$ that interest us. Their properties.

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1.3. **Random variables and expectations.**

D&S Chapter 4.1-2,4.7

We will discuss expectations at an earlier stage than do D&S, then go back to discussion of topics that are taken up in chapters 2 and 3 of D&S.

1.3.1. *Expectation of a random variable.*

1.3.2. *Expectation from probability, or vice versa?*

1.3.3. *Conditional expectation.*

The law of iterated expectations. Conditional probability from conditional expectation. Conditioning on sets, random variables, and σ-fields.

1.3.4. *Independence.*

1.3.5. *Conditional and marginal probabilities from joint densities or distribution functions.*

1.3.6. *Bayes' rule.*

1.3.7. *Transformations of random variables: Jacobians.*

1.3.8. *Density functions for sums and quantiles of random variables.*

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1.4. **Moments, quantiles.**

D&S Chapter 4.3-6

1.4.1. *Characterizing the shape of a distribution.*

The moment sequence and quantiles as descriptive devices.

1.4.2. *Covariance and correlation; eigenvector/eigenvalue decomposition.*

1.4.3. *The characteristic function.*

1.4.4. *Markov and Chebyshev inequalities.*

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1.5. **Particular distributions.**

D&S Chapter 5.1-6,5.9-12,7.1-4

1.5.1. Γ, *Beta.*

1.5.2. *Multivariate Normal.*

The density function. Normality of conditionals and of linear combinations. Equivalence of independence and lack of correlation.

1.5.3. χ^2, *t*, *Wishart, inverse-Wishart, inverse-*Γ.

2. Statistics

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2.1. **Models, parameters, nuisance parameters, error terms, shocks.**

D&S Chapter 6.1-4, 6.7-9,7.6

2.1.1. *Making data analysis more "objective" by defining models, parameters, and shocks.*

2.1.2. *Likelihood, sufficiency.*

2.1.3. *Applying Bayes rule: priors.*

Conjugate; ignorance.

2.1.4. *Posteriors for multivariate normal distributions with conjugate priors.*

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2.2. **Central limit theorems, asymptotic approximations.**

D&S 5.7

2.2.1. *Asymptotic gaussian shape of the likelihood.*

2.2.2. *Asymptotic distribution of functions of averages: the* Δ *method.*

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2.3. **Estimation, competing models, tests, confidence intervals.**

D&S 6.5-6,7.5,7.7,8.1,8.5-9

2.3.1. *Bayesian posterior means and medians.*

2.3.2. *Unbiased estimation.*

2.3.3. *Maximum likelihood.*

2.3.4. *Forming and using posteriors across models.*

Schwarz and Akaike criteria; LaPlace approximation.

2.3.5. *Tests and confidence intervals.*

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2.4. **Computer-intensive analysis of posteriors.**

D&S 11.1-4

References

DeGroot, M. H., and M. Schervish (2002): *Probability and Statistics*. Addison-Wesley, Reading, Massachusetts, third edn.

Ferguson, T. S. (1967): *Mathematical Statistics: A Decision Theoretic Approach*. Academic Press, New York and London.

Gelman, A., J. B. Carlin, H. S. Stern, and D. B. Rubin (1995): *Bayesian Data Analysis*. Chapman and Hall, London.

Schervish, M. J. (1995): *Theory of Statistics*, Springer Series in Statistics. Springer, New York.