LECTURE 6: Γ, BETA, INFERENCE FOR THE PARAMETERS OF THE NORMAL

1. GAMMA

pdf:	$\dfrac{x^{p-1}e^{-x}}{\Gamma(p)}$ or $\dfrac{\alpha^p x^{p-1}e^{-\alpha x}}{\Gamma(p)}$
cdf:	incomplete gamma function
moments:	$EX = \dfrac{p}{\alpha} \quad \text{Var}(X) = \dfrac{p}{\alpha^2} \quad \text{mode} = \dfrac{p-1}{\alpha}$
sums:	X, Y independent, $X \sim \Gamma(p), Y \sim \Gamma(q)$
	$\Rightarrow X + Y \sim \Gamma(p+q)$
notes:	$p > 0, \alpha > 0, \quad p$ integer $\Rightarrow \Gamma(p) = (p-1)!$

2. GAMMA CONT.

This is a waiting time distribution. If events occur at arbitrary moments in time, with the time from one event to the next independent of previous times between events, and if the expected number of events per unit time is α, then $\Gamma(p,\alpha)$ is the distribution of time from any date t_0 until p more events have occurred.

Closure under sums does not extend to arbitrary linear combinations, or to sums that mix scale parameters α.

3. BETA

pdf:	$x^{p-1}(1-x)^{q-1}\dfrac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)}$
cdf:	incomplete Beta function
moments:	$EX = \dfrac{p}{p+q} \quad \text{mode} = \dfrac{p-1}{p+q-1}$
note:	$p, q > 0$
note:	X, Y independent, $X \sim \Gamma(p), Y \sim \Gamma(q)$
	$\Rightarrow \dfrac{X}{X+Y} \sim \text{Beta}(p,q)$

4. Gamma pdf's