EXERCISE DUE TUESDAY, 9/23

(1) Suppose the df of (X,Y) on the unit square is $\frac{1}{2}\big(xy+\min\{x,y\}\big)$.
   (a) How does the df behave *outside* the unit square?
   (b) What are the marginal df's of X and Y?
   (c) What is the df of (U,V), where $U=\log X$, $V=\log Y$?
   (d) Does the joint distribution of X,Y have a density w.r.t. Lebesgue measure? Why or why not? What about the marginal distributions of X and Y? Of U and V?
   (e) Find the conditional expectations of $Y \mid X$, $U \mid V$, and $V \mid U$.

(2) The random variables X and Y are jointly $N(0,I)$, i.e. they have pdf

$$\frac{1}{2\pi}e^{-\frac{1}{2}(x^2+y^2)}.$$

Suppose we transform to polar coordinates, $\rho=\sqrt{X^2+Y^2}$, $\theta=\arctan(Y/X)$.
   (a) What is the joint pdf of ρ and θ?
   (b) What is their joint cdf?
   (c) What is the conditional pdf of $\rho \mid \{\theta=\pi/2\}$?
   (d) Suppose the joint distribution of X,Y were that of a joint normal restricted to the upper half-plane (i.e., $Y\geq 0$). What would be the conditional pdf of $Y \mid \{X=0\}$? Explain why this is different from the conditional pdf of $\rho \mid \{\theta=\pi/2\}$.