Econ. 517, Econometric Theory I Fall 2001 Chris Sims

Explanation of 2c

Problem 2c asks you to explain why no Bayesian estimator for λ derived from a symmetric loss function could ever be zero with positive probability, assuming that $\lambda = 0$ does not have prior probability 1.

A student has pointed out that this is not true without qualification. First, it should be pointed out that "symmetric loss function" was meant to imply $L(\lambda - \hat{\lambda}) = L(\hat{\lambda} - \lambda)$ and, since this is an estimation problem (meaning the losses should increase with $|\lambda - \hat{\lambda}|$), $L'(\hat{\lambda} - \lambda) > 0$ for $\hat{\lambda} > \lambda$. But these conditions are not enough to guarantee the result. For example, if

$$L(|\lambda - \hat{\lambda}|) = \begin{cases} |\lambda - \hat{\lambda}| & |\lambda - \hat{\lambda}| \leq 1 \\ .05\,|\hat{\lambda} - \lambda| + .95 & |\lambda - \hat{\lambda}| > 1\,, \end{cases}$$

and if the posterior pdf puts probability .5 on $\lambda = 0$ and .5 on $\lambda = 2$, then expected losses are minimized at $\hat{\lambda} = 0$. To make the assertion in the problem statement true, we need to add a further regularity condition on the loss function. For example, it is true if the loss function is convex or if it is differentiable at $\lambda = \hat{\lambda}$ (in which case, by the symmetry, its derivative is zero at that point).