

Matrix decompositions and the 517 problem set due 10/9 The problem using matrix decomposition should take only a few minutes once you've entered the matrices you're asked to analyze and applied the `chol` and `eig` commands. The interpretations you're asked to do shouldn't require you to go beyond straightforward application of the lecture notes.

However, the notes and lectures didn't make it explicit that in the decomposition $\Sigma = Q'DQ$ the matrix Q' is the matrix of right eigenvectors of Σ , with the columns normalized to unit length, and the diagonal of D contains the eigenvalues of Σ on the diagonal. If your linear algebra is rusty, this might not have been obvious. Once you see this, you can get Q and D with the Matlab line

```
[v,d]=eig(sigma);
```

This will give you Q' as v and D as d . Then you form $Q'D^{1/2}$ and use it to represent X in terms of an independent normal Z vector as in the notes and lecture. Answering the question requires only constructing such representations and commenting on whether they account for most variation in X with just a few elements of Z .

My impression from discussions with Hong is that quite a few students haven't seen these matrix decompositions before and are finding them mysterious. Being able confidently to use mathematical results even when they seem mysterious is a useful skill. The results that you need, as stated in the notes, are that it is always possible to write a positive definite matrix Σ as $W'W$, with two possible forms for W (among others): W can be chosen upper triangular, or it can be chosen to satisfy $W = D^{1/2}Q$ with $Q'Q = I$ and D diagonal with non-negative diagonal elements. You don't have to understand how to prove these results or how to do the calculations by hand in order to use the results that Matlab will compute for you. You can check that `chol` finds an upper triangular W and `eig` gives you a Q and D of the required types with $Q'DQ = \Sigma$.

For those of you who haven't seen the results before, or don't remember them, Hong will discuss them during part of the next precept. Also, since apparently some people have been completely buffaloed by this problem, it will be all right, assuming you are one of these people, to hand in that problem on Thursday instead of Tuesday.