Econ. 517, Econometric Theory I Fall 2001 Chris Sims

Exercise*

(1) Prove that for a random vector X with the standard $N(0, \Sigma)$ pdf, Σ is in fact the covariance matrix of X. [Suggestion: First prove it in the univariate case, then in the case of independent X, then in the general case, by writing a general X as $W'Z$ where Z is independent.]

(2) Construct an example of a three-dimensional random vector X with the property that each of the three possible two-element subvectors (X_i, X_j) of X has a marginal distribution that is $N(0, I)$, but the three X's considered jointly are not independent. [Though this is in some sense simple, it may be hard to see how to approach it. Don't waste a lot of time on it if it seems impossible.]

(3) Using a Taylor approximation of the log of the pdf about its maximum, construct normal approximations to the $\Gamma(1.5)$, $\Gamma(2)$, and $\Gamma(4)$ pdf's. In each case, plot both the original pdf and the normal approximation to it. Do this also for $\Gamma(.5)$, though in this case, since the pdf does not have a peak, you should make the Taylor expansion about the mean (which is .5) and, because this is not the peak, you will have a first-order as well as a second-order term.

(4) Here are two Σ matrices:

$$\begin{bmatrix} 1.1000 & 0.1000 & 0.1000 & 1.0000 \\ 0.1000 & 1.1000 & 1.0000 & 0.1000 \\ 0.1000 & 1.0000 & 1.1000 & 0.1000 \\ 1.0000 & 0.1000 & 0.1000 & 1.1000 \end{bmatrix} \begin{bmatrix} 1.0000 & 0.5000 & 0.3333 & 0.2500 \\ 0.5000 & 0.2750 & 0.1667 & 0.1250 \\ 0.3333 & 0.1667 & 0.1222 & 0.0833 \\ 0.2500 & 0.1250 & 0.0833 & 0.0688 \end{bmatrix}.$$

For each, compute both an eigenvalue decomposition and a Choleski decomposition. Do both methods "work" to suggest structure for the matrix? Do they suggest similar definitions of the important "Z" components explaining variation in the X's with these covariance matrices? Matlab commands relevant here are `chol` and `eig`.