Volatility-identified SVAR robustness

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Main themes of the talk

- The model supporting structural VAR's identified through time-varying heteroskedasticty (volatility-identified SVAR's) is a natural and convenient extension of the standard reduced form VAR framework, even when the structural interpretation of the model disturbances is not the center of interest.

- Identification through volatility may seem to be fragile, relying on strong and disputable assumptions. But in fact the shapes of impulse responses estimated this way are consistent under a wide range of deviations from these assumptions.

Related literature

- Brunnermeier, Palia, Sastry, and Sims (2018), 10-variable, monthly US VAR, ID'd through volatility.

- Gourieroux and Monfort (2014), recognized possibility of identification through non-Gaussianity. (What does this have to do with ID through volatility? We'll see.)

- Sims (2020), written before I was aware of Gourieroux and Monfort's paper. G & M suggest methods that work with any form of non-Gaussianity. This paper of mine shows that assuming t-distributed errors delivers conistency even if the truth is fat-tailed non-Gaussianity of other forms.

The SVAR model

$$A(L)y_t = \varepsilon_t$$

$$\varepsilon_t \sim N(0, I) \text{ and i.i.d. across } t\,.$$

We'll assume ε_t and the innovation vector u_t spann the same space, i.e. no invertibility issue. (G&M show that modeling non-Gaussianity can lead to identification even in the presence of non-invertibility.)

Identification: As is well known, there are too many free parameters in $A(L)$. Various approaches have been taken to impose enough restrictions on the system that it is identified: Restrictions on A_0, restrictions on $A(1)$, "external instruments", "narrative".

Time-varying volatility

- In macroeconomic and financial time series models, large outliers and obvious variations in the level of volatility of disturbances are pervasive.

- In the US, the 1970's, with oil crises rising inflation, and active monetary policy were more volatile than the post-1983 "Great Moderation", and Volckers 1979-82 reserve targeting policy created great volatility in interest rates. 2009, and now the pandemic, are again generating volatility spikes.

- Time-varying volatility is a staple component of empirical modeling of asset prices and returns.

Why worry about varying volatility?

- If we are using a VAR for forecasting, we get consistent estimates of the best linear predictor and correct asymptotic distribution theory so long as we have finite variance disturbances.
- But ignoring time varying volatility can have a substantial cost in forecasting performance when it is extreme.
- Large individual outliers can dominate the fit.
- Entering in to a period of high volatility, the large outliers will lead the fitting algorithm to make large revisions in the estimated linear structure, precisely when this is the wrong thing to do.

How to model varying volatility?

- In a moderately large SVAR model like our 10-variable model in BPSS, allowing all 55 free parameters in the residual covariance matrix to change over time in an unrestriced way is an impractically large sacrifice of degrees of freedom, particularly for a forecasting model.

- The model underlying a volatility-identified SVAR is one approach to parsimoniously modeling this time variation;

$$\mathrm{Var}(u_t) = B\Lambda_t B' \,,$$

  with B fixed and Λ_t diagonal. In our 10-variable case, the additional parameters from every change in the covariance matrix of innovations is 10, instead of 55.

Identification through volatility

- In the standard SVAR setup the innovation covariance matrix is given by

$$\mathrm{Var}(u_t) = A_0^{-1}(A_0^{-1})' \,.$$

- If we allow for time variation in the variances of the structural disturbances, we modify the $\mathrm{Var}(\varepsilon_t) = I$ assumption to make it $\mathrm{Var}(\varepsilon_t) = \Lambda_t$, with Λ_t diagonal.

- Now suppose that for two different times, or spans of time, we can estimate the reduced form covariance matrix Σ_i, $i = 1, 2$.

Identification through volatility (2)

Under the assumption that the variation through time in Σ arises from variation in structural shock variances,

$$\Sigma_i = A_0^{-1}\Lambda_i(A_0^{-1})'$$
$$\therefore \Sigma_1^{-1}\Sigma_2 = A_0'\Lambda_1^{-1}\Lambda_2(A_0^{-1})' \,.$$

$\Lambda_1^{-1}\Lambda_2$ is diagonal, so this last expression is the Jordan canonical form. If all the elements of $\Lambda_0^{-1}\Lambda_1$ are distinct, the eigenvectors of the matrix, which are the columns of $(A_0^{-1})'$, are uniquely determined.

Implications of this result

- In a structural VAR, once you have allowed for time variation in structural shock variances, it is possible that you need no other restrictions on the system to achieve identification.

- In BPSS, we began by using restrictions on A_0 for identificatioin, realized we had to allow for time varying volatility to get credible likelihood-based inference, and then discovered that our identification remained strong even with no restrictions on A_0.

- Of course even when identification through volatility alone is in principle possible, it may be weak, so that additional well-founded restrictions can still be useful.

Summary to this point

- There are two ways to arrive at a volatility-identified SVAR specification.

  1. Begin with a descriptive or forecasting model objective, notice that volatility of residuals in fact varies widely, model it parsimoniously with the $\Sigma = B\Lambda_t B'$ form.
  2. Begin with a structural, causal modeling objective, notice that volatility of structural residuals in fact varies widely, model that variation, and discover that other identifying restrictions are no longer needed.

How to model the variation in Λ_t

- Identify sub-periods in your sample with distinct relative variances of shocks, ideally lining up the dates of transition with known shifts in sources of variation in the economy.

- Use a hidden Markov chain specification (as in **?**) to estimate endogenously the dates of regime switches.

- Model drift, rather than discrete shifts, in the structural shock variances.

We'll focus on the first of these, which is what is in BPSS.

What could go wrong?

- We might have the dates of the changes in Σ wrong. What then?
- There might actually be little time variation in volatility.
- The usual asymptotic theory, and inference based on the Gaussian likelihood, can be a bad approximation if the residuals are strongly non-normal.
- It might be that the apparent time variation in volatility actually reflects time variation in $A(L)$ itself.

The last of these is a serious issue, and we will not take it up here. The others, we will see, are less of a problem than it might appear.

Guessing wrong about regime shift dates

- Bayesian estimation based on the Gaussian likelihood is efficiently done by recognizing that, conditional on A_0, the system can be represented as a system of weighted least squares regression equations, with disturbances unrelated across equations. The weights depend on the sequence of Λ_t's.

- As in any weighted least squares equation, in these the estimated coefficients converge to the same limiting value in large samples, whether the weights are "correct" or not.

- And A_0 itself is correctly pinned down for a given set of Σ_t's, because the average of $u_t u_t'$ across the period will have the form $A_0^{-1}\Lambda_i(A_0^{-1})'$ even if the actual relative variances of structural shocks vary over the sub-period.

Non-normality

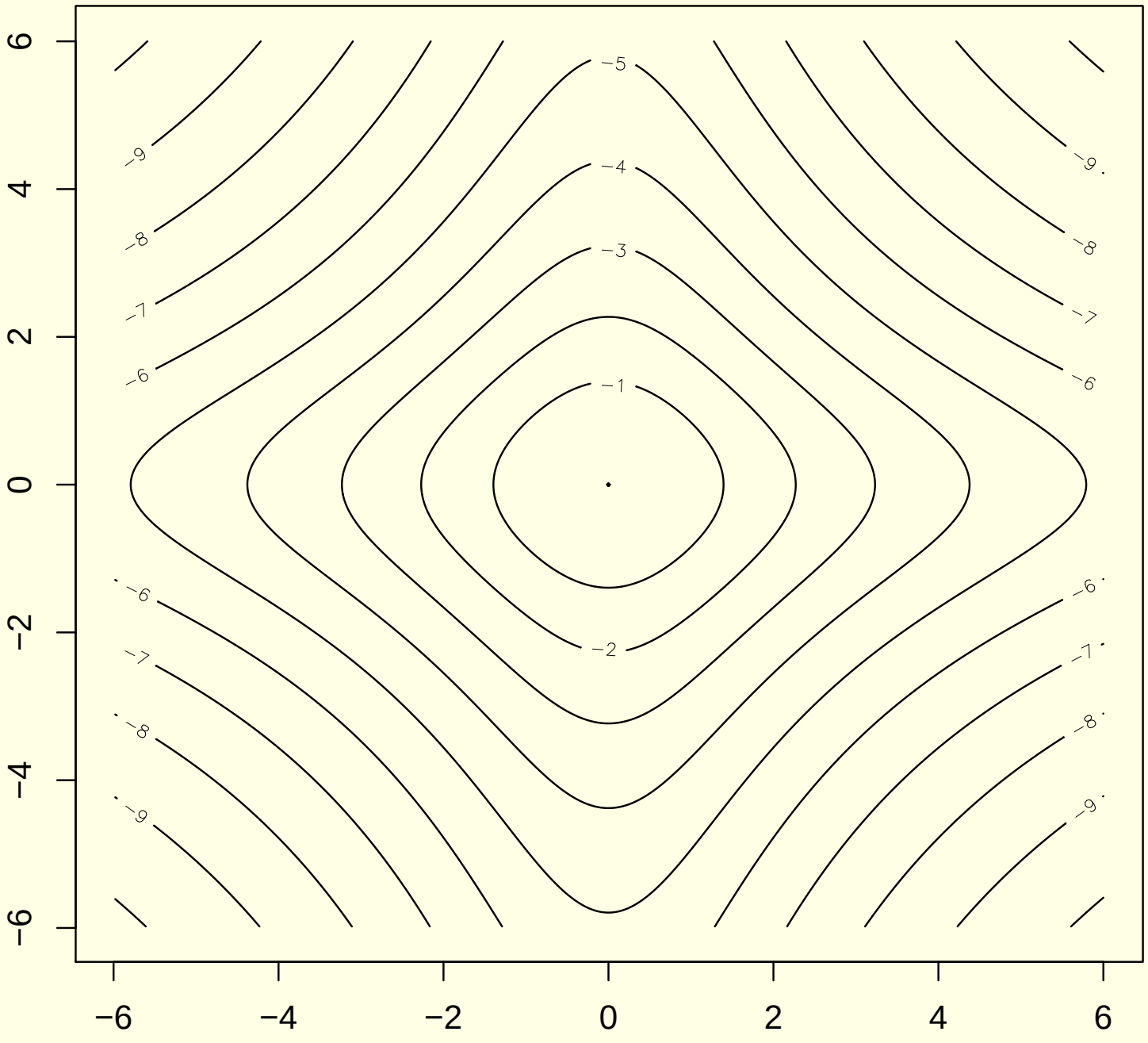
- G&M show that non-normality of structural shocks, assuming they are independent, is an aid to identification.

- In BPSS we estimate assuming t-distributed shocks, with each structural shock a scaled t variate with 5.7 degrees of freedom. This is clearly restrictive, and indeed we found even with this specification there were a few too many large outliers to match the distributional assumption.

- But if the shocks have a fat-tailed, symmetric distribution, likelihood-based estimation assuming the t distribution is consistent in general with no further identifying assumptions.

Why assuming independent t's is robust

log likelihood contours for independent t, df=3



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References

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