Econ. 513, Time Series Econometrics Fall 1999 Chris Sims

Take-Home Final Exam*

I. Suppose

$$y(t) = \begin{bmatrix} 0.3 & -0.2 \\ -0.3 & 0.9 \end{bmatrix} y(t-1) + \varepsilon(t)$$

and $\varepsilon(t) \mid \{y(s), s < t\}] \sim N(0, \Omega)$, for all t.

(a) Prove that y is stationary.

(b) Prove that, if $\{A_s, s = 0, \ldots, \infty\}$ are the coefficients in the fundamental MAR representation of y in terms of its innovations, the following sequence $\{A_s^*\}$ of three matrices are *not* the first three elements of the A_s sequence:

$$A_0^* = I \quad A_1^* = \begin{bmatrix} 0.3 & -0.2 \\ -0.3 & 0.9 \end{bmatrix} \quad A_2^* = \begin{bmatrix} 0.15 & -0.24 \\ -0.30 & 0.87 \end{bmatrix}$$

II. Consider the model

$$y(t) = c + \rho_1 y(t-1) + \rho_2 y(t-2) + \varepsilon(t) \,,$$

in which $\varepsilon(t) \mid \{y(t-s),\ s > 0\} \sim N(0, \sigma^2)$. Applying least squares (i.e. maximum likelihood conditional on initial observations) to the model, we obtain the estimates $\hat\rho_1 = 1.9$, $\hat\rho_2 = -.9024$, $\hat c = 3$, $\hat\sigma^2 = .2$. It can be shown that if these estimates were the parameters generating the y process, we would have, unconditionally, $\mathrm{Var}(y(t)) = 427.1820$, $\mathrm{Cov}(y(t), y(t-1)) = 426.6431$.

(a) Suppose the initial conditions that we have treated as fixed in the OLS estimation are $y(-1) = 1.7$, $y(0) = 1.1$. Show that, according to the estimated model, these initial conditions are implausible if they are thought to have been generated from the same model as the later observations.

(b) Construct a dummy observation that implies that if lagged y's are set at the sample mean of the initial observations, the model should predict that current y is also near the mean of the initial observations, with a standard deviation on this dummy observation of $.2\sigma^2$, where this σ^2 is the variance of the equation disturbances as defined above. You are expected to give numerical values for all elements of the dummy observation.

(c) Explain why adding this dummy observation to the sample is likely to reduce the extent to which the estimated parameters imply the initial conditions are implausible.

III. Consider the model

$$\underset{2\times 1}{y(t)} = Ay(t-1) + \varepsilon(t) \,,$$

in which as usual we assume $\varepsilon(t)$ to be the innovation in y. We assume to keep the problem simple $\varepsilon(t) \mid \{y(s),\, s < t\} \sim N(0, I)$, with no uncertainty about this covariance matrix. We have OLS estimates conditional on initial observations in which we emerge with

$$\hat{A} = \begin{bmatrix} .7 & .2 \\ .2 & .7 \end{bmatrix} , \quad \frac{\hat{u}'\hat{u}}{T} = I \, ,$$

where the $T \times 2$ $\hat{u}$ matrix is the OLS residuals. The sample second-moment matrix of the right-hand side variables (what we usually label "$X'X$") is

$$\begin{bmatrix} 30 & 20 \\ 20 & 30 \end{bmatrix} .$$

When the second equation alone is estimated with a_{21} constrained to zero, it produces $\hat{a}_{22} = .9$ and $\hat{u}_2'\hat{u}_2/T = 1.2$. We have a prior pdf over A that is Gaussian with mean $\bar{A} = I$ and covariance matrix

$$I \otimes \begin{bmatrix} .5 & -.4 \\ -.4 & .5 \end{bmatrix}$$

conditionally on the parameter space with a_{22} unconstrained. Conditionally on the hypothesis that $a_{21} = 0$, our prior mean is the same and our prior covariance matrix is

$$\begin{bmatrix} .5 & -.4 & 0 \\ -.4 & .5 & 0 \\ 0 & 0 & .25 \end{bmatrix} ,$$

where the upper left 2×2 block corresponds to the coefficients of the first equation and the lower right to the single unconstrained coefficient on the second equation. Our prior puts equal probability on the constrained and unconstrained versions of the model. What is the posterior probability that the constraint is true? [Note: This problem involves a lot of algebra and numerical calculation. Be sure you have explained how you are setting up the calculation and why, so that if you run out of time or make algebra or arithmetic mistakes you can get partial credit.]

IV. (a) If $y(t) = 10y(t-1) + \varepsilon(t)$, with ε i.i.d. across time and distributed as $N(0, 1)$, and if $y(t)$ is stationary, find an expression for the innovation in y as a function of current and past y and find the variance of the innovation.

(b) If the estimated time path of coefficients from the Kalman smoother is in fact much less smooth then the estimated time path of coefficients, using the same data, based on the Kalman filter, does this mean you have made a mistake in calculation? Does it mean the model is wrong?

(c) If the spectral density of y shows a single very sharp peak at frequency π, then y has strong seasonal fluctuations, and we do not have to know whether the time unit is a month, a quarter, a day, etc. in order to assert this. Is this true or false? Why?