Econ. 556b Spring 1999 Chris Sims

Exercise on Checking for Fundamental Representations

1. In each of the following models, determine whether it is possible that ε_t is the innovation in y_t (or at least spans the same space) and (at the same time) y_t is stationary:
   (a) $y_t = 1.8y_{t-1} - .9y_{t-2} + \varepsilon_t$
   (b) $y_t = 1.9y_{t-1} - .9y_{t-2} + \varepsilon_t$
   (c) $2y_t = 1.95y_{t-1} - .9y_{t-2} + \varepsilon_t$
   (d) $y_t = \varepsilon_t - 1.9\varepsilon_{t-1} + .9\varepsilon_{t-2}$
   (e) $y_t = .9\varepsilon_t - 1.9\varepsilon t - 1 + \varepsilon_{t-2}$
2. Suppose

$$y_t = \varepsilon_t + \begin{bmatrix} 1.2 & -.4 \\ -.4 & 1.2 \end{bmatrix} \varepsilon_{t-1} \,, \tag{1}$$

with ε_t i.i.d. $N(0, I)$. Show that if y is a stationary process, then it can also be represented as

$$y_t = \nu_t + \begin{bmatrix} .9375 & .3125 \\ .3125 & .9375 \end{bmatrix} \nu_{t-1} \,, \tag{2}$$

with $\nu(t)$ i.i.d. $N(0, \Sigma)$ and

$$\Sigma = \begin{bmatrix} 1.6 & -.96 \\ -.96 & 1.6 \end{bmatrix} \,. \tag{3}$$

Because these processes are all normally distributed, you can show that they are the same processes by showing that they imply the same $R_y(s) = \text{Cov}(y_t, y_{t-s})$ function.
3. Show that neither of the two MAR's in 2 are fundamental — i.e. that neither ε nor ν is the innovation in y.
4. Find the fundamental MAR for the y process in the preceding problem. [Extra credit. This will require that you compute an eigenvalue decomposition of the coefficient matrix (the Jordan decomposition works fine here), replace unstable roots with their inverses, and revise the Σ matrix to reproduce the R_y function. Along the way you will need to solve a version of the Ricatti equation $\Sigma + C\Sigma C' = V$ for Σ. There is a special matlab routine to do this, called `lyap.m`, which may or may not be available on the department machines. In this 2×2 case, though, the equation is just a system of 3 equations in three unknowns (because of symmetry of Σ) and therefore can be solved directly. I think it likely that no one in the class will get this right, so don't spend more than a couple of hours working on it.]