Answer to Bayesian Practice Exercise

The log likelihood for the sample is

$$\ell(\beta,\sigma;y,x) = \sum_{j=1}^{6}\left\{\left(\frac{(y_j-x_j\beta)^2}{2\sigma^2}-\log(\sigma\sqrt{(}2\pi))\right)\mathcal{I}(y_j>0)+\log\left(\Phi\left(\frac{-x_j\beta}{\sigma}\right)\right)\mathcal{I}(y_j=0)\right\}. \quad (1)$$

Once this is in hand, the rest of the assignment is just a matter of getting the computer to draw the contours. A program to compute the log likelihood in matlab is listed below. It is set up to take vector arguments and to return the whole matrix of values of ℓ for different β's and σ's in a single function call.

After the program listing, there are contour plots for the three cases in the problem set and for an additional case, with $y = [0, 0, 0.3760, 0.8630, 0, 3.5727]$, that was generated as a random draw from a model with $\beta = 0$.

Program listing

```
function [lhnaive,lh,llh,mle,mler,mlec]=TobitEx(y,x,b,sigma)
%function [lhnaive,lh,llh,mle,mler,mlec]=TobitEx(y,x,b,sigma)
[B,S]=meshgrid(b,sigma);
ny=length(y);
u=(repmat(y,[1 size(B)])-repmat(x,[1,size(B)]).*rrepmat(B,ny,1))...
      ./rrepmat(S,ny,1);
zindex=find(y==0);
nzindex=find(y~=0);
nz=length(zindex);
nnz=length(nzindex);
llh=sum(-.5*u(nzindex,:,:).^2)-sum(log(rrepmat(S,nnz,1))-.5*log(2*pi));
uz=u(zindex,:,:);
llh2=sum(log(max(.5*(erf(uz/sqrt(2))+1),1e-200)));
llh=squeeze(llh+llh2);
[mx,ix]=max(llh);
[mle,mlec]=max(mx);
mler=ix(mlec);
lh=exp(llh-mle);
lhnaive=sum(-.5*u.^2)-sum(log(rrepmat(S,6,1)))-.5*log(2*pi);
mlen=max(max(lhnaive));
lhnaive=exp(squeeze(lhnaive)-mlen);
```

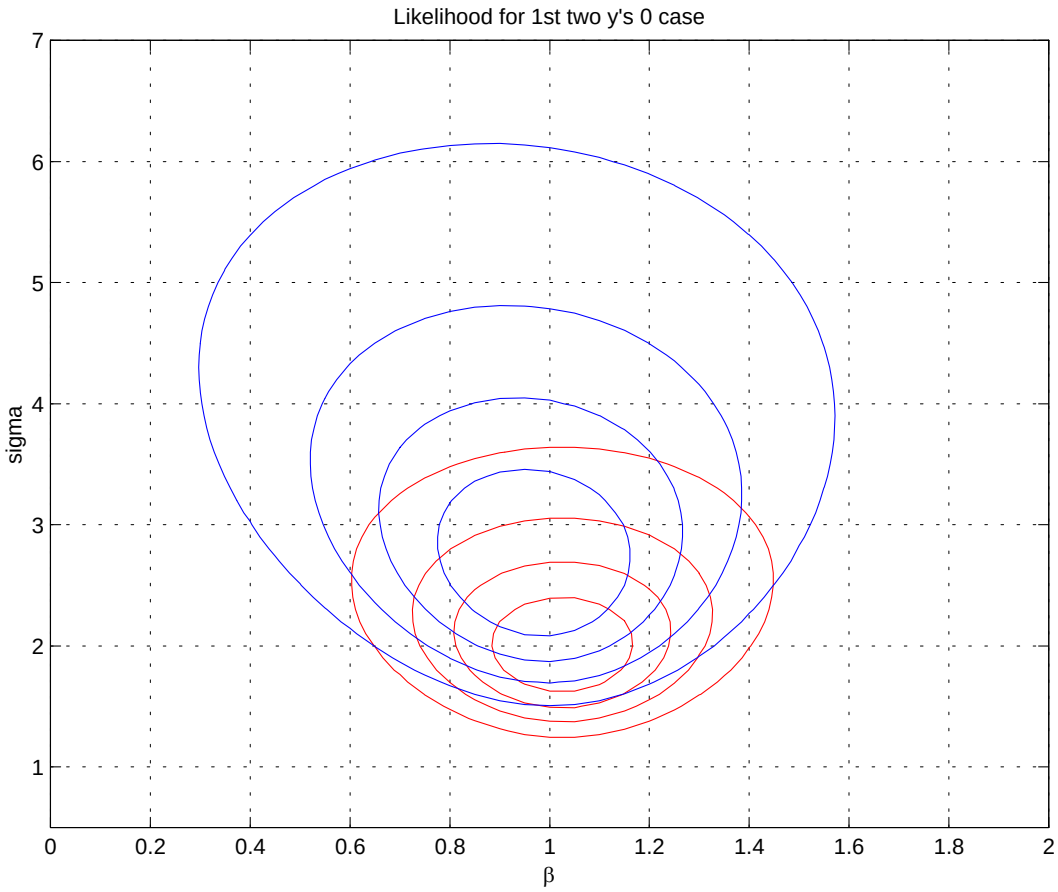

FIGURE 1. Likelihood b (red is naive OLS likelihood)

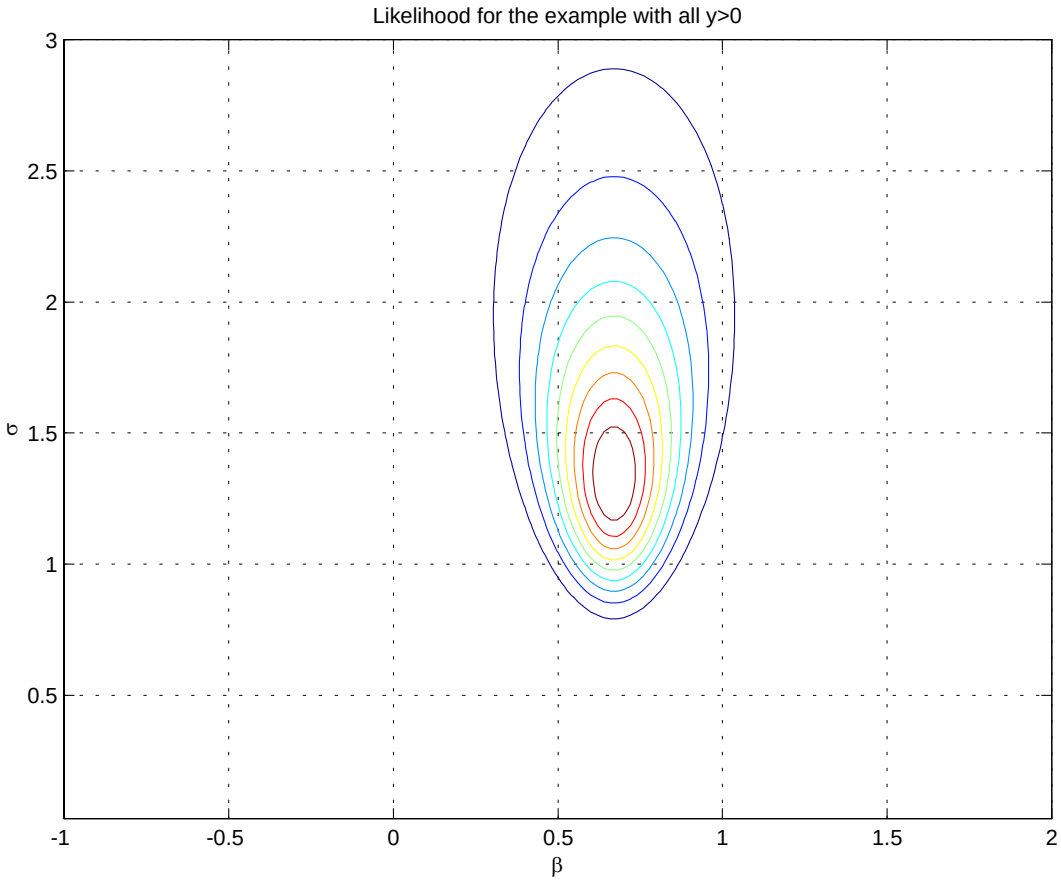

FIGURE 2. Likelihood c

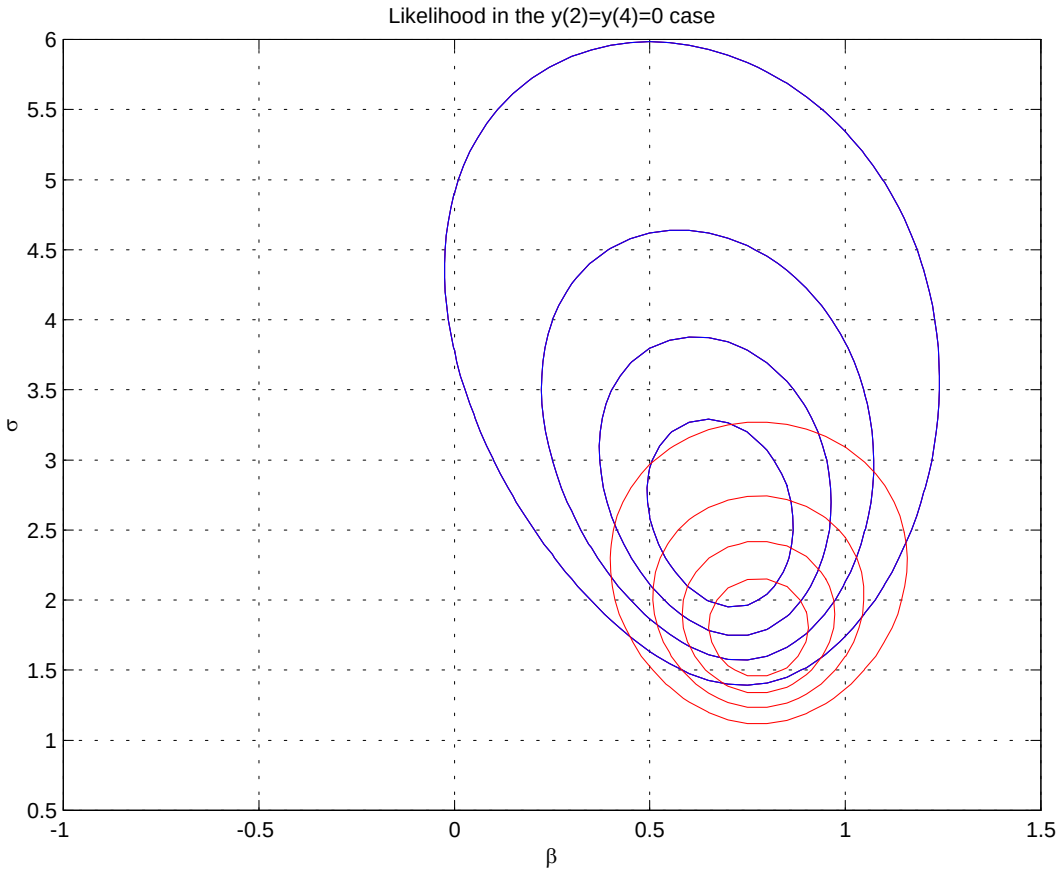

FIGURE 3. Likelihood d, (red is naive OLS likelihood)

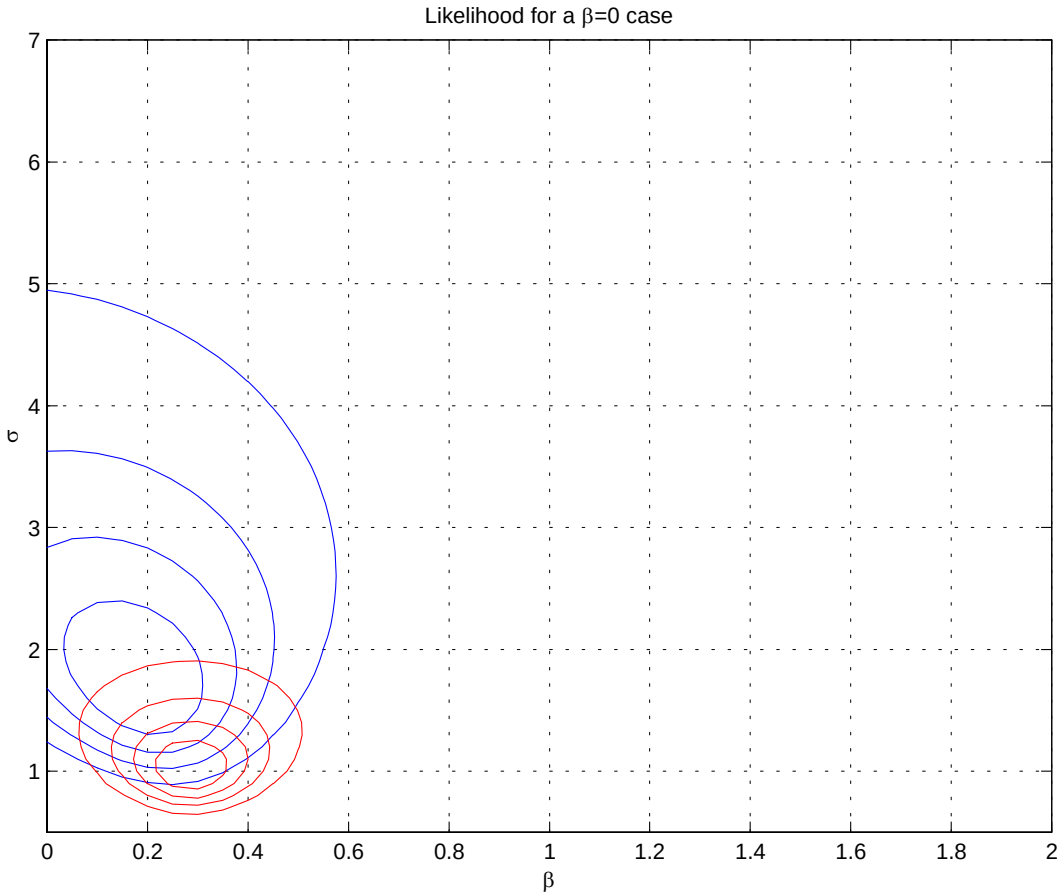

FIGURE 4. Likelihood for β=0 sample