Econ. 556b Spring 1999 Chris Sims

Problem Set: Kalman Smoothing

The Natural Rate. Consider the model

$$\Delta\pi_t = \alpha_t \Delta\pi_{t-1} + \beta_t U_{t-1} + \gamma_t + \varepsilon_t \tag{1}$$

$$\begin{bmatrix}\alpha_t\\ \beta_t\\ \gamma_t\end{bmatrix} = \begin{bmatrix}\alpha_{t-1}\\ \beta_{t-1}\\ \gamma_{t-1}\end{bmatrix} + \zeta_t\ , \tag{2}$$

with the elements of the vector $\eta_t = [\varepsilon_t\ \zeta_t']'$ mutually independent and independent of all variables in the system dated before t. We also assume joint normality for η. The variable π is the inflation rate (change in log of GDP deflator) and U is the (adult civilian) unemployment rate. The symbol Δ is the first difference operator, $\Delta x_t = x_t - x_{t-1}$. At least if the variances of the elements of ζ_t are small enough, it is reasonable to interpret $-\gamma_t/\beta_t$ as the "natural rate of unemployment" the rate that is consistent with nonaccelerating inflation in the long run.[1]

You are to use this model to form, first, Kalman filtered estimates of the time series of parameters $[\alpha, \beta, \gamma]$, then smoothed estimates of them based on the full time series. Use them to construct two (filtered and smoothed) time series of estimates of the natural rate. Use a prior that sets

$$\begin{bmatrix}\alpha_0\\ \beta_0\\ \gamma_0\end{bmatrix} \sim N\left(\begin{bmatrix}.9\\ -.1\\ .5\end{bmatrix}, \begin{bmatrix}.04 & .01 & -.01\\ .01 & .01 & -.01\\ -.01 & -.01 & .09\end{bmatrix}\right) \tag{3}$$

Make the variance matrix of ζ_t in (2) $\delta = .004$ times the variance matrix in (3). Set σ_ε^2, the variance of ε_t in (1), equal to the sample variance of $\Delta\pi_t$.

Because this is a regression model, not a complete model of the data, you will need to use as likelihood the p.d.f. of the data conditional on all values of U and on the initial value of π.

Data are available on the course web page in `smoothMData.xls` in MS Excel format. They are also available in text format, ready to be read in to Matlab, in `smthdat.txt`. The text file has no labels, so it can be read in easily. You need to know that it runs from 1948:1 to 1998:2, quarterly, and that its three columns are unemployment rate in per cent, GDP deflator (chain index form), and real GDP. (For this exercise you don't use the third column.) Of course you need to convert the price index data to inflation rate form. Set $\pi_t = 400 \cdot \log(P_t/P_{t-1})$ to get it in units of per cent at annual rates. Plot the natural rate series you find, together with error bands for them. Since the natural rate is a ratio of two parameters that are both normal, it has a complicated distribution.

[1] Our use of this model in an exercise implies no endorsement of it as substantive economics. It does not "work" at all in most European economies, and even in the US it tends to lead to "wrong" signs—implying high unemployment produces high inflation—if the π variable is allowed to enter in non-differenced form.

If the parameters γ_t and β_t are estimated with high precision at each t (very unlikely), you can use the approximation $\mathrm{Var}((\mu+\varepsilon)/(\lambda+\eta)) = \sigma_\varepsilon^2/\lambda^2 + \mu^2\sigma_\eta^2/\lambda^4$. Otherwise, you will need to use Monte Carlo simulation to estimate the standard errors or quantiles of the distributions to get the error bands.

Finally, check the sensitivity of your results to the degree of time variation, indexed by the parameter δ above, and to σ_ε by redoing the work for $\delta = .0001$ and $\delta = .0016$ and with σ_ε also both four times larger and one fourth as large. [This is four additional cases. You don't need to trace out all 8 possible combinations of these variations in δ and σ_ε^2 . Consider not only how the results change, but also how the size of the posterior p.d.f. changes in assessing whether it appears that it would be a good idea (which you are not supposed to pursue here) to proceed to formal Monte Carlo integration over a prior on one or both of these parameters.