Econ. 556b Spring 1999 Chris Sims

Take-Home Exam

1. You have estimated by OLS a reduced-form VAR, i.e. a model of the form

$$\underset{m\times 1}{y(t)} = \sum_{s=1}^{k} B(s)y(t-s) + c + \varepsilon(t),\ t = 1,\dots,T\,, \tag{1}$$

with lag length $k = 5$, number of variables $m = 6$, and sample size $T = 100$. Here we interpret T as the number of observations used in OLS estimation, so that data are actually available on an additional k "initial condition" $y(t)$'s at the beginning of the sample. From the OLS estimates $\hat{B}(s)$ and $\hat{c}$ of $B(s)$ and c, you form the $T \times m$ matrix $\hat{\varepsilon}$ whose t'th row is

$$\hat{\varepsilon}(t) = y(t) - \sum_{s=1}^{k} \hat{B}(s)y(t-s) - \hat{c} \tag{2}$$

We define $S = \hat{\varepsilon}'\hat{\varepsilon}$. You also have estimated the system subject to the constraint $B_{ij}(s) = 0$ for all $i > q$, $j <= q$. You have used OLS for this also, which means that you have simply used a shorter list of right-hand-side variables in applying OLS to the last $m - q$ equations in the system. We use $\hat{\varepsilon}_R$ as notation for the estimated residuals from this restricted system, and the $m\times m$ matrix $S_R = \hat{\varepsilon}_R'\hat{\varepsilon}_R$.

(a) Describe how to use S and S_R to form an asymptotically justified classical χ^2 test at the 5% level of significance for the null hypothesis that the full set of restrictions is true. Explain how, if at all, the classical validity of the test depends on whether unit roots are present in the model. [No proofs or details of how the test depends on the presence of unit roots are expected.]

(b) Explain why the same test statistic you used in 1a can be interpreted as making an approximate statement about the shape of the posterior pdf on the parameter space under a flat prior. What exactly is the subset of the parameter space that has approximately 5% probability under this flat-prior Bayesian interpretation of the χ^2 statistic?

(c) Explain how the χ^2 statistic could be used as part of a formula for determining the approximate posterior probability that the restrictions are true if the prior put non-zero probability on the subspace where the restrictions are true. [A precise answer would depend on the details of the form of the prior. You do not need to specify these details. Just assume there is a prior that puts positive probability on the restricted subspace, has a density over the remainder of the space, and has a density within the restricted space.]

(d) How, if at all, does the validity of the Bayesian asymptotic approximations you used in your answers to 1b and 1c depend on whether unit roots are present in the model?

(e) How can you check whether in your sample the Bayesian asymptotic approximations are accurate? How can you check whether in your sample the classical asymptotic approximations are accurate?

2. Consider the following model, which might be interpreted as one in which recession results from inefficiencies accumulated during periods of high output.

$$y(t) = \gamma(t) + \varepsilon(t) \tag{3}$$

$$\gamma(t) = \begin{cases} \gamma_0 & \text{whenever } S(t) < 0 \\ \gamma_1 & \text{whenever } S(t) \geq 0 \end{cases} \tag{4}$$

$$S(t) = -y(t) + \rho S(t-1) + \nu(t)\,. \tag{5}$$

We can interpret y as the output growth rate, γ_0 as the mean recession growth rate, and γ_1 as the mean non-recession growth rate. We suppose $\varepsilon(t)$ and $\nu(t)$ are independent of each other and i.i.d. normal across time. We have data only on $y(t)$, $t = 1, \ldots, T$. Suggest a Markov chain Monte Carlo scheme for estimating the parameters of this model.