Handling low frequencies and initial conditions

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Implausible fit of deterministic components

- AR models, particularly VAR models or models with many lags, if estimated by methods that condition on initial observations (like OLS), tend to imply that $E_0[y_t]$, $t = 1, \ldots, T$, where $t = 1$ is the start of the sample on the left-hand-side variable, is an implausibly accurate predictor of the trend or long-run swings in the sample $y_1, \ldots, y_T$

- This happens because the criterion of fit applies no penalty to parameter values that make the initial conditions highly implausible as draws from the model's implied unconditional distribution for y_t. The model then attributes the low-frequency behavior of the data to a process, lasting through much or all of the sample, of slow return to "normalcy" from these exotic initial conditions.

Is this a problem?

- We may believe that the initial conditions are not reasonably modeled as having been generated by this same model, running for a long time.

- For example, a VAR for German macro data, with the first year of data 1950. Initial conditions were unusual, and dynamics arising from a return to a mean or trend are plausible.

- But usually an estimated model that implies initial conditions are far out in the tail of the unconditional distribution are implausible.

- The problem decreases, in a certain sense (see below) as sample size increases, but in a panel, where many VAR's are fit across countries, say, the false claim that initial conditions and parameters are independent remains a problem as N (number of countries) becomes large.

Why worse as models get bigger?

- In a univariate, one-lag model, return-to-trend dynamics can only take the exponential form $(y_0 - Ey)\rho^t$.

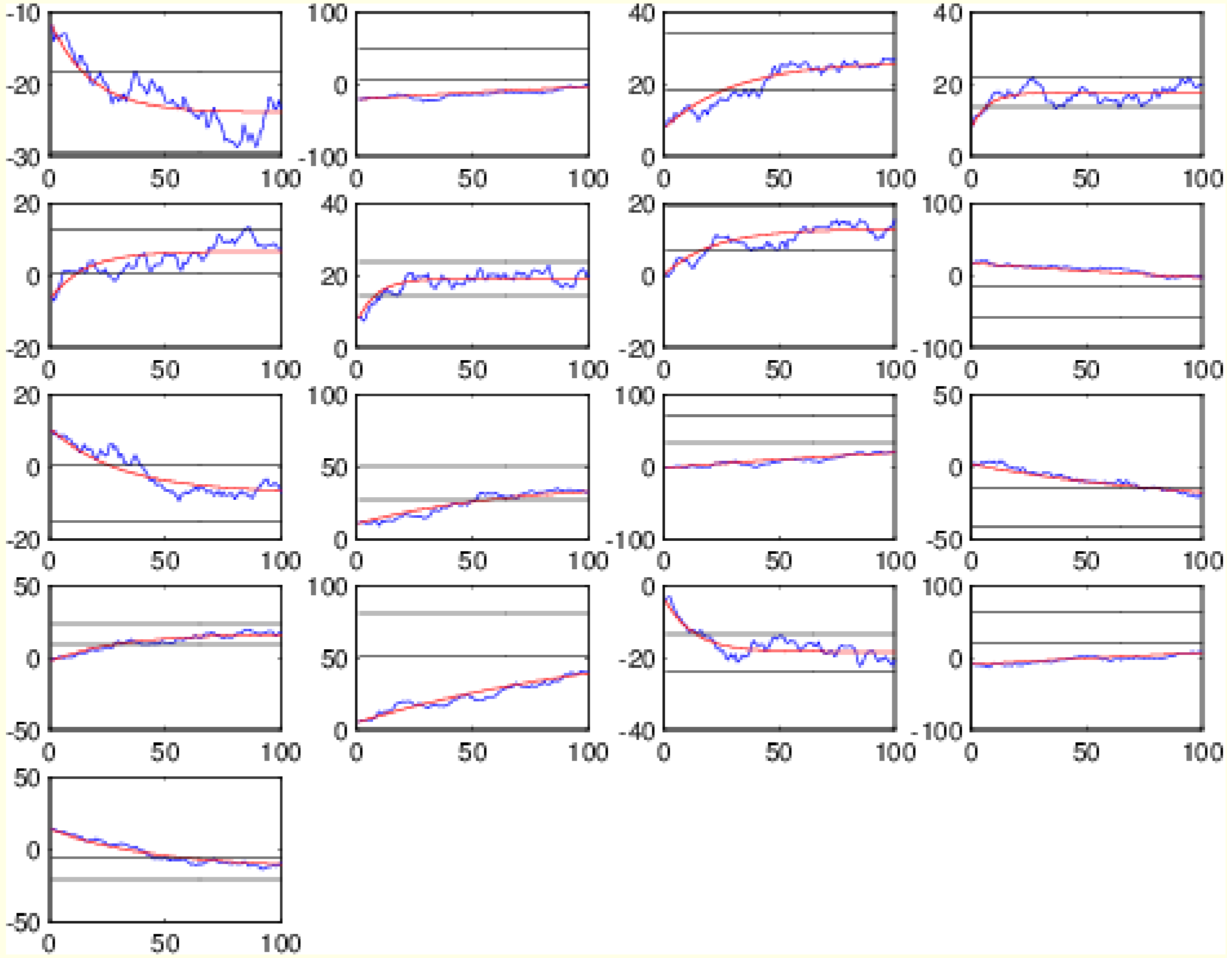
- With k lags, a univariate model can produce return-to-trend dynamics that are linear combinations of k exponentials. In particular, if all the observations (including the initial k observations) like on a k'th order polynomial, the AR can predict them perfectly.

- A VAR with k lags on n variables has kn roots and can fit perfectly an arbitrary collection of kn'th order polynomials (assuming the polynomials are linearly independent).

- So the potential for implausibly precise forecasts from initial conditions grows rapidly with n and k, and indeed in practice the problem is clearly worse in larger models.

Remedies

- At least check for the problem: Use estimated coefficient values to construct $E_0 y_t$, plot these against actual data to see if the results make sense.
- Use the distribution of initial conditions in estimation.
- Use a prior that captures the idea that implausibly precise long run forecasts have low prior probability.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Modeling trends

- See Sims (revised 1996).
- Applied statisticians and macroeconomists often treat low frequency variation as a nuisance, like seasonal variation.
- The idea then is to get rid of it in a way that leaves inference about the remainder of the variation minimally affected.
- In some cases — when there is a clean separation of low and high frequency variation — a variety of different methods to “detrend” may give similar results.

- Take out linear or log-linear determinstic trend, first or second difference, Hodrick-Prescott filter, e.g.

Typical spectral shape

- With seasonality, there often is a clean separation of seasonal and non-seasonal variation.
- Separating "trend" from macroeconomic business cycle variation is much less clear.
- Granger's "typical spectral shape".

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References

SIMS, C. A. (revised 1996): "Inference for Multivariate Time Series with Trend," Discussion paper, presented at the 1992 American Statistical Association Meetings, http://sims.princeton.edu/yftp/trends/ASAPAPER.pdf, http://sims.princeton.edu/yftp/trends/ASAPAPER.pdf.