SVAR’S

1. SVAR’S VS. DSGE’S

- Linearized DSGE’s are SVAR’s if all states are observable.
- Even if not, they are SVARMA’s, and ARMA’s can be approximated arbitrarily well by AR’s.
- But usually SVAR’s are more loosely restricted, aiming at identifying policy behavior, or some single shock or group of shocks, without producing a full behavioral interpretation.
- Two approaches to such sets of minimal identifying restrictions have been common (leaving DSGE-based approaches aside): Restrictions on $A_0$ and long run restrictions.

2. INVERTIBILITY

- A linearized DSGE will imply that the observable vector $y_t$ satisfies
  $$A(L; \theta) y_t = B(L; \theta) \varepsilon_t$$
  where $\varepsilon_t$ is i.i.d. $N(0, I)$ and, to normalize, we assume $A_0 = I$.
- The theory need not imply that $\varepsilon_t$ is recoverable from current and past values of $y_t$ — i.e. it need not imply that the model is invertible.
- The common approaches to SVAR identification ignore this possibility. They assume that $\varepsilon_t$ is a linear combination of the reduced form innovations $u_t$.

3. INVERTIBILITY II

- Invertibility fails whenever $\varepsilon_t$ is longer than $u_t$, which seems likely to be always, in principle.
- It is easy to construct theoretical examples where invertibility fails.
- This is not as serious a problem as it seems: We need only approximate invertibility.
- Approximate invertibility holds when the projection of the shock we are interested in (e.g. the monetary policy behavior shock) on current and past $y$ produces a high $R^2$.
- We can get usually get good approximate invertibility if we are sure to include in $y$ variables that respond promptly to the structural shock we are interested in (e.g., interest rates for the monetary policy shock).
4. CHECKING APPROXIMATE INVERTIBILITY

A straightforward method: Usually the linearized DSGE has the form
\[ w_t = Gw_{t-1} + H\varepsilon_t \]
\[ y_t = Hw_t \]

Also usually \( H \) is full column rank, so that if we know \( w_t \) and \( w_{t-1} \) we can recover \( \varepsilon_t \) exactly —
\[ \text{Var}(\varepsilon_t \mid t) = \Theta \text{Var}(w_t \mid t) \Theta' \]

Starting from any initial variance matrix for \( w \), the Kalman filter delivers a sequence of \( \text{Var}(w_t \mid t) \) matrices that do not depend on the \( y_t \) sequence and that usually converge. Check whether the above expression converges to zero for those elements of the \( \varepsilon_t \) vector that matter. (Sims and Zha, *Macroeconomic Dynamics* 2006).

5. SVAR IDENTIFICATION


**SVAR:**

\[ A(L)y_t = \varepsilon_t. \]

(ignoring the possibility of a constant or exogenous variables).

Reduced form:

\[ (I - B(L))y_t = u_t, \quad \text{Var}(u_t) = \Sigma, \]

where \( A_0u_t = \varepsilon_t \), therefore \( A_0^{-1}(A_0^{-1})' = \Sigma \), and \( A_0(I - B(L)) = A(L) \).

The RF fully characterizes the probability model. The SVAR has more parameters than the RF, so there is an id problem. (There could be an id problem even if the parameter count matched; the SVAR might restrict the probability model for the data even if it had more parameters than the RF.)

6. LONG RUN RESTRICTIONS: BLANCHARD AND QUAH

7. RESTRICTIONS ON \( A_0 \)

If the SVAR restrictions are on \( A_0 \) alone and leave \( A_0 \) invertible, they leave \( B(L) = -A_0^{-1}A^+ \) unrestricted. The log likelihood can be written as
\[ \frac{T}{\log |A_0|} - \frac{1}{2} \text{trace}(A_0'A_0) \sum_{i=1}^{T} \hat{u}_i \hat{u}_i', \]

where \( \hat{u}_i = (I - \hat{B}(L))y_t \) are the least-squares residuals. Thus if the restrictions are on \( A_0 \) alone,

- Likelihood maximization is OLS, followed by nonlinear maximization on \( A_0 \) alone.
Posterior simulation can be done in blocks, with the $B$ block a simple draw from a multivariate normal.

8. Extensions by RWZ

- They show a straightforward method for checking global identification. (Hamilton had shown a local id check.)
- They show that certain kinds of nonlinear restrictions (e.g. on impulse responses) can also be handled with their approach.
- They claim that the nonlinear maximization can be done faster in identified cases by searching explicitly for the rotation of the Choleski decomposition of the $RF\Sigma$ that satisfies the restrictions.

9. The cases for exact id 0-restrictions in a 3D system

\[
\begin{bmatrix}
  x & x & x \\
  0 & 0 & 0 \\
  x & x & x \\
\end{bmatrix}
\quad \text{or} \quad
\begin{bmatrix}
  0 & x & x \\
  0 & x & x \\
  0 & x & x \\
\end{bmatrix}
\Rightarrow \text{incomplete}
\]

\[
\begin{bmatrix}
  x & x & x \\
  0 & x & x \\
  0 & 0 & x \\
\end{bmatrix}
\Rightarrow \text{identified}
\]

\[
\begin{bmatrix}
  x & x & 0 \\
  x & 0 & x \\
  0 & x & x \\
\end{bmatrix}
\Rightarrow \text{local exact id, global overid, and unid}
\]

\[
\begin{bmatrix}
  x & x & 0 \\
  0 & x & x \\
  0 & x & x \\
\end{bmatrix}
\Rightarrow \text{not identified, but first equation is overid’d}
\]

\[
\begin{bmatrix}
  x & 0 & 0 \\
  0 & x & x \\
  x & x & x \\
\end{bmatrix}
\Rightarrow \text{identified, but adding a restriction can undo id}
\]

10. Typical contemporaneous ID for money

$r$, fast block $y$, slow block $z$:

\[
\begin{bmatrix}
  x & ? & 0 \\
  x & x & x \\
  0 & 0 & x \\
\end{bmatrix}
\]

11. Block triangular normalization

**Thm:** Linear transformations of the equations of a system can always make it triangular with an identity covariance matrix.