SVAR'S

1. SVAR's vs. DSGE's

- Linearized DSGE's *are* SVAR's if all states are observable.
- Even if not, they are SVARMA's, and ARMA's can be approximated arbitrarily well by AR's.
- But usually SVAR's are more loosely restricted, aiming at identifying policy behavior, or some single shock or group of shocks, without producing a full behavioral interpretation.
- Two approaches to such sets of minimal identfying restrictions have been common (leaving DSGE-based approaches aside): Restrictions on A_0 and long run restrictions.

2. Invertibility

- A linearized DSGE will imply that the observable vector y_t satisfies

$$A(L;\theta)y_t = B(L;\theta)\varepsilon_t$$

  where ε_t is i.i.d. $N(0,I)$ and, to normalize, we assume $A_0 = I$.
- The theory need not imply that ε_t is recoverable from current and past values of y_t — i.e. it need not imply that the model is invertible.
- The common approaches to SVAR identification ignore this possibility. They assume that ε_t is a linear combination of the reduced form innovations u_t.

3. Invertibility II

- Invertibility fails whenever ε_t is longer than u_t, which seems likely to be always, in principle.
- It is easy to construct theoretical examples where invertibility fails.
- This is not as serious a problem as it seems: We need only approximate invertibility.
- Approximate invertibility holds when the projection of the shock we are interested in (e.g. the monetary policy behavior shock) on current and past y produces a high R^2.
- We can get usually get good approximate invertibility if we are sure to include in y variables that respond promptly to the structural shock we are interested in (e.g., interest rates for the monetary policy shock).

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4. CHECKING APPROXIMATE INVERTIBILITY

A straightforward method: Usually the linearized DSGE has the form

$$w_t = G w_{t-1} + H\varepsilon_t$$
$$y_t = H w_t$$

Also usually H is full column rank, so that if we know w_t and w_{t-1} we can recover ε_t exactly —

$$\mathrm{Var}(\varepsilon_t \mid t) = \Theta \,\mathrm{Var}(w_t \mid t)\Theta'$$

Starting from any initial variance matrix for w, the Kalman filter delivers a sequence of $\mathrm{Var}(w_t \mid t)$ matrices that do not depend on the y_t sequence and that usually converge. Check whether the above expression converges to zero for those elements of the ε_t vector that matter. (Sims and Zha, *Macroeconomic Dynamics* 2006).

5. SVAR IDENTIFICATION

Complete reference: Rubio-Ramirez, Waggoner and Zha, "Structural Vector Autoregressions: Theory of Identification and Algorithms for Inference". On Rubio-Ramirez Duke website.

SVAR:

$$A(L)y_t = \varepsilon_t \,.$$

(ignoring the possibility of a constant or exogenous variables).

Reduced form:

$$(I - B(L))y_t = u_t \,, \qquad \mathrm{Var}(u_t) = \Sigma \,,$$

where $A_0 u_t = \varepsilon_t$, therefore $A_0^{-1}(A_0^{-1})' = \Sigma$, and $A_0(I - B(L)) = A(L)$.

The RF fully characterizes the probability model. The SVAR has more parameters than the RF, so there is an id problem. (There could be an id problem even if the parameter count matched; the SVAR might restrict the probability model for the data even if it had more parameters than the RF.)

6. LONG RUN RESTRICTIONS: BLANCHARD AND QUAH

7. RESTRICTIONS ON A_0

If the SVAR restrictions are on A_0 alone and leave A_0 invertible, they leave $B(L) = -A_0^{-1}A^+$ unrestricted. The log likelihood can be written as

$$\frac{T}{\log}\,|A_0| - \tfrac{1}{2}\,\mathrm{trace}(A_0' A_0)\sum_{t=1}^{T} \hat{u}_t \hat{u}_t' \,,$$

where $\hat{u}_t = (I - \hat{B}(L))y_t$ are the least-squares residuals. Thus if the restrictions are on A_0 alone,

- Likelihood maximization is OLS, followed by nonlinear maximization on A_0 alone.

- Posterior simulation can be done in blocks, with the B block a simple draw from a multivariate normal.

8. Extensions by RWZ

- They show a straightforward method for checking global identification. (Hamilton had shown a local id check.)
- They show that certain kinds of nonlinear restrictions (e.g. on impulse responses) can also be handled with their approach.
- They claim that the nonlinear maximization can be done faster in identified cases by searching explicitly for the rotation of the Choleski decomposition of the RF Σ that satisfies the restrictions.

9. The cases for exact id 0-restrictions in a 3D system

$$\begin{bmatrix} x & x & x \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix} \text{ or } \begin{bmatrix} 0 & x & x \\ 0 & x & x \\ 0 & x & x \end{bmatrix} \Rightarrow \text{incomplete}$$

$$\begin{bmatrix} x & x & x \\ 0 & x & x \\ 0 & 0 & x \end{bmatrix} \Rightarrow \text{identified}$$

$$\begin{bmatrix} x & x & 0 \\ x & 0 & x \\ 0 & x & x \end{bmatrix} \Rightarrow \text{local exact id, global overid, } \textit{and} \text{ unid}$$

$$\begin{bmatrix} x & x & 0 \\ 0 & x & x \\ 0 & x & x \end{bmatrix} \Rightarrow \text{not identified, but first equation is overid'd}$$

$$\begin{bmatrix} x & 0 & 0 \\ 0 & x & x \\ x & x & x \end{bmatrix} \Rightarrow \text{identified, but } \textit{adding} \text{ a restriction can undo id}$$

10. Typical contemporaneous ID for money

r, fast block y, slow block z:

$$\begin{bmatrix} x & ? & 0 \\ x & x & x \\ 0 & 0 & x \end{bmatrix}$$

11. Block triangular normalization

Thm: Linear transformations of the equations of a system can always make it triangular with an identity covariance matrix.