EXERCISE ON WOLD DECOMPOSITION, ARMA MODELS

(1) In all of the models below, $\varepsilon_t$ is i.i.d. $N(0,1)$ and $y_t$ is a stationary process. Though they have different coefficients, not all of them define different distributions for $y_t$. Which pairs define identical stochastic processes?

\[
y_t = .7y_{t-1} + \varepsilon_t - .9\varepsilon_{t-1} \tag{1.1}
\]
\[
y_t = 2.9y_{t-1} - 2.8y_{t-2} + .9y_{t-3} + \varepsilon_t - 2\varepsilon_{t-1} + \varepsilon_{t-2} \tag{1.2}
\]
\[
y_t = .9y_{t-1} + \varepsilon_t - .7\varepsilon_{t-1} \tag{1.3}
\]
\[
y_t = 1.7y_{t-1} - .95y_{t-2} + .175y_{t-3} + \varepsilon_t - 1.9\varepsilon_{t-1} + 1.15\varepsilon_{t-2} - .225\varepsilon_{t-3} \tag{1.4}
\]
\[
y_t = .9y_{t-1} + \varepsilon_t \tag{1.5}
\]

(2) The stochastic process $y$ is a finite-order moving average of the form

\[
y_t = \varepsilon_t + 1.7\varepsilon_{t-1} + .7\varepsilon_{t-1}
\]

with $\varepsilon_t$ i.i.d. $N(0,1)$.

(a) Show that this process does not have an AR representation, even if we allow infinitely many lags.

(b) Determine the form of the best predictors of $y_t$ based on 2, 5, and 10 lagged values of $y$ and compare the forecast error variance of these approximations to the forecast error variance of $E[y_t \mid \{y_s, s = -\infty, \ldots, t - 1\}]$. [You’ll need to use a computer. The toeplitz functions in R and Matlab should be helpful.]

(3) Again all the models below have i.i.d. $N(0, I)$ $\varepsilon$ processes, and in each, $y$ is stationary. For each model, find the fundamental MAR and compare the variance of the innovation to the variance of $\varepsilon$.

\[
y_t = \varepsilon_t + 2\varepsilon_{t-1} + 1.1\varepsilon_{t-2} \tag{3.1}
\]
\[
y_t = \varepsilon_t - 2\varepsilon_{t-1} + .99\varepsilon_{t-2} \tag{3.2}
\]
\[
y_t = \varepsilon_t + \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} \varepsilon_{t-1} \tag{3.3}
\]

In the first two of these, $y$ is scalar, while in the last, both $y$ and $\varepsilon$ are two-dimensional.

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