TAKE-HOME FINAL EXAM

Answer all three questions. They are equally weighted in the grading.

(1) Consider the system

$$y_t = \begin{bmatrix} 1.1 & 0.4 & 0 \\ 1.8 & -0.3 & 0 \\ 0.5 & 0.5 & 1 \end{bmatrix} y_{t-1} + \begin{bmatrix} -.2 & -.3 & 0 \\ -.3 & -.2 & 0 \\ -.5 & -.5 & 0 \end{bmatrix} y_{t-2} + \varepsilon_t \tag{1}$$

where the first two elements of y_t are wages of workers 18-45 years old and 46-65 years old, while the last element is the consumer price index, all measured in logs.

(a) Does this system (under the usual assumption that it is an AR representation with the ε_t's a stationary innovation process) imply non-stationarity of any elements of y? Which ones?

(b) Does this system imply that there is cointegration present? If so, find the cointegrating vector or vectors.

(c) What does the system imply about the long run behavior of the relative wages of young and old and the long run behavior of real wages?

(2) (a) For a stationary reduced form vector autoregression with finite-variance residuals, error bands for impulse responses computed by Bayesian posterior simulation under normality assumptions are asymptotically justified as frequentist confidence intervals. Explain in what sense this is true and explain why it is true.

(b) Is it equally true that error bands for impulse responses in an *overidentified structural* VAR are asymptotically justified as frequentist confidence intervals? Why or why not?

(3) Suppose we are considering the following dynamic factor model:

$$\underset{n\times 1}{y(t)} = \underset{n\times 1}{\alpha} z(t) + \underset{n\times n}{\rho}\, y(t-1) + \underset{n\times 1}{\varepsilon(t)}\,, \tag{2}$$

where $\varepsilon_i(t) \sim N(0, \sigma_i^2)$, each i, independently across i and t and is independent of the entire z process and of all $y_i(s)$'s dated $s < t$. We would like to allow for the possibility that the vector α (but not the other parameters) shifts randomly over time, according to a Markov switching process that is independent of z and ε.

(a) Write out the likelihood function for this model. Consider how to include initial conditions in the likelihood formulation.

(b) Discuss whether there are normalization issues in this model, and how you might take care of them.

(c) Discuss whether proper priors on some or all parameters are required to make MCMC iteration converge for this model.

(d) Describe in detail an MCMC iteration scheme that would allow inference on this model.

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