HANDLING LOW FREQUENCIES AND INITIAL CONDITIONS

1. IMPLAUSIBLE FIT OF DETERMINISTIC COMPONENTS

- AR models, particularly VAR models or models with many lags, if estimated by methods that condition on initial observations (like OLS), tend to imply that $E_0[y_t]$, $t = 1, \ldots, T$, where $t = 1$ is the start of the sample on the left-hand-side variable, is an implausibly accurate predictor of the trend or long-run swings in the sample $y_1, \ldots, y_T$
- This happens because the criterion of fit applies no penalty to parameter values that make the initial conditions highly implausible as draws from the model's implied unconditional distribution for y_t. The model then attributes the low-frequency behavior of the data to a process, lasting through much or all of the sample, of slow return to "normalcy" from these exotic initial conditions.
- There is no logical contradiction here, but usually parameter values with these characteristics are not plausible.

2. WHY WORSE AS MODELS GET BIGGER?

- In a univariate, one-lag model, return-to-trend dynamics can only take the exponential form $(y_0 - Ey)\rho^t$.
- With k lags, a univariate model can produce return-to-trend dynamics that are linear combinations of k exponentials. In particular, if all the observations (including the initial k observations) like on a k'th order polynomial, the AR can predict them perfectly.
- A VAR with k lags on n variables has kn roots and can fit perfectly an arbitrary collection of kn'th order polynomials.
- So the potential for implausibly precise forecasts from initial conditions grows rapidly with n and k, and indeed in practice the problem is clearly worse in larger models.

3. REMEDIES

- At least check for the problem: Use estimated coefficient values to construct $E_0 y_t$, plot these against actual data to see if the results make sense.
- Use the distribution of initial conditions in estimation.
- Use a prior that captures the idea that implausibly precise long run forecasts have low prior probability.

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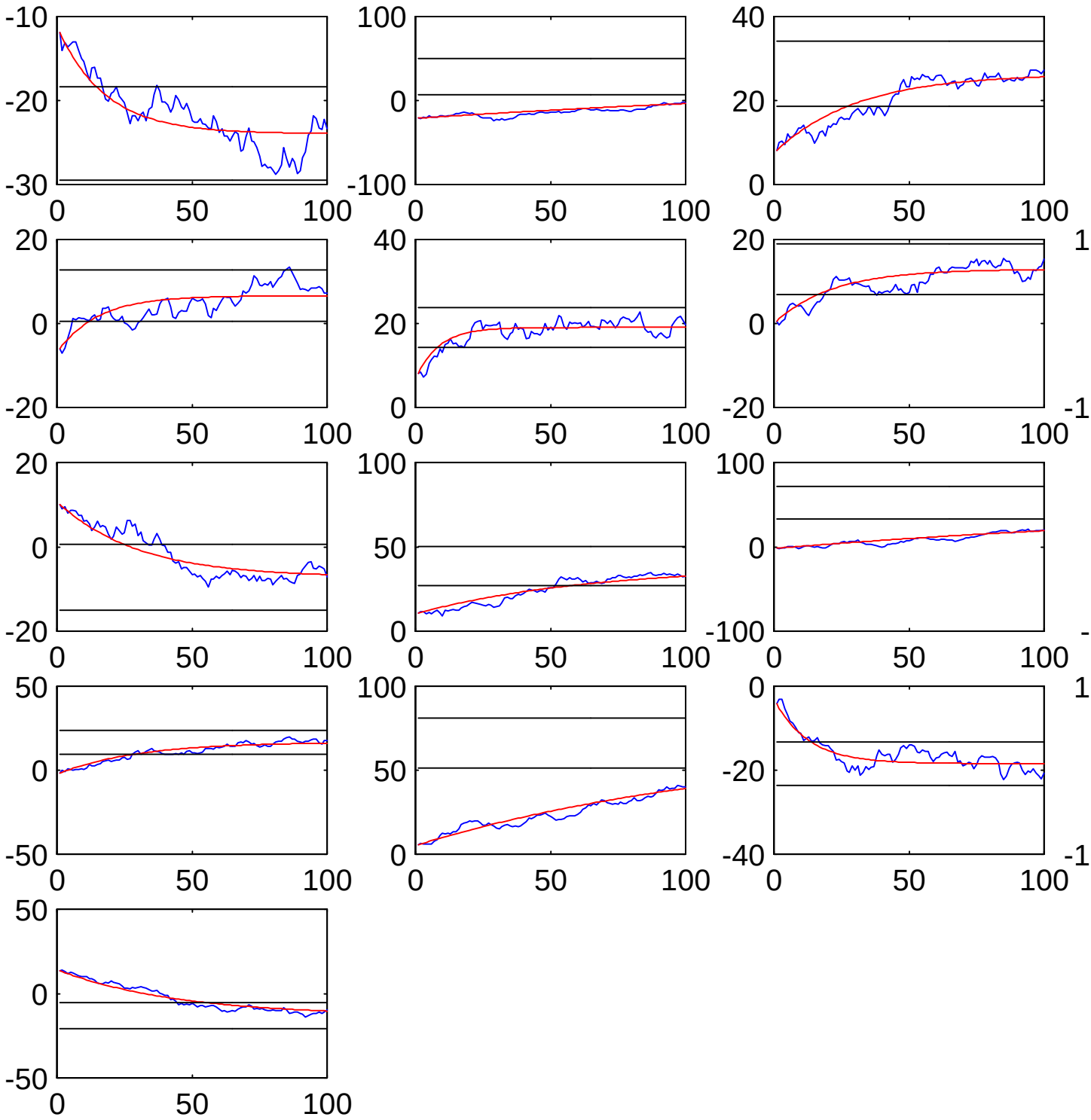
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