VAR ESTIMATION EXERCISE

Using the same data set as we used for our previous Kalman filter exercise (`ppi3` — three components of the producer price index), estimate a six-lag reduced form VAR model, including a constant term.

(a) Compute and plot orthogonalized impulse responses for the system. Orthogonalize using the symmetric square root of the covariance matrix, the Cholesky decomposition, and the Cholesky decomposition with variable order reversed. Are there notable differences in the impulse responses among the various approaches to orthogonalization? In plotting, make three graphs, with responses of one variable to each of the three shocks on each graph, with the lines distinguished by color or line type.

(b) For the Cholesky decomposition, compute and plot the impulse responses with 68% error bands. This will require generating many draws from the posterior on the coefficients, computing responses for each draw of the covariance matrix of residuals and regression coefficients, and computing and locating, for each time horizon, the 16%, 84%, and median quantiles. (For 68% bands even 100 will be enough, but you can use more to get smoother graphs if you have a fast computer.) To avoid too many lines on a graph, these should be in 9 separate plots.

See the plots at the end for these first two parts. Since crude materials prices are set in auction markets, we expect them to be rather unpredictable. It is not surprising, therefore, that the response of crude materials prices to own shocks is much bigger than its response to the other two shocks when crm is first in the ordering. The plot with error bands of crm's response to fin shows that at almost all horizons zero is in the 68% bands. The one or two months with zero outside the bands have it still probably inside bands of double the size (which would be approximately 95% bands.) However the response of crm to int, though small, is clearly positive at many time horizons with high probability.

(c) Construct a Wald test of the hypothesis that crude materials prices are GCP to the other two series. Also of the hypothesis that there is an ordering of all three variables by GCP, with crude materials first, intermediates second, and finished goods last.

This requires forming the sample covariance matrix of residuals, taking its crossproduct with what `rfvar3` labels `xxi` (the inverse of the $X'X$ matrix), and picking off the rows and columns corresponding to the coefficients on lagged int and fin in the crm equation (which are indexed by 2,3, 5,6, 8,8, 11,12, 14,15, 17,18) in `rfvar3`'s `xxi`. The chi-squared statistic, with 12 d.f., is 23.77, which is in the 97.8% tail of the chi-square(12) distribution. For the stricter hypothesis of a causal ordering, these same coefficients must be zero, and in addition those on lagged fin in the int equation (indexed by 22, 25, 28, 31, 34, 37) This produces a chi-squared statistic of 46.2, which is in the .9997 tail of a chi-squared(18). So both restrictions are rejected at conventional significance levels. The Schwarz criterion, which we will discuss soon, is an asymptotic approximation to a Bayesian odds ratio that balances numbers of parameters with fit. Here it would compare these chi-squared statistics to 79 and 119, respectively: in other words it would accept both restrictions.

(d) Does it appear to you that the unobservable component model you fit to the same data in the previous exercise is in conflict with this VAR model? [A really good answer to this question might calculate the impulse responses implied by the previous exercise model to compare to this VAR's impulse responses, but that is a major task, and we have not discussed in detail how to do it.]

The previous model gave each series an idiosyncratic component independent of the other idiosyncratic components and of the common component $\bar{p}$. It therefore apparently implies that to get good estimates of past $\bar{p}$, which would be needed for forecasting, one would want to use all three series, and thus no series would be GCP to the others. On the other hand, our actual estimates showed that finished goods prices were nearly identical to $\bar{p}$ through much of the sample. If they were exactly the same, then finished good prices would be GCP to the other series (because the model implies $\bar{p}$ is best forecast by its own past, with no input from the observable series if past $\bar{p}$ is observable). However, what we find with the estimated VAR is that each of the variables has a "significant" response to shocks in at least one other variable. (This is true in both the crm-int-fin ordering and the fin-int-crm ordering.) So the GCP ordering implied by fin$= \bar{p}$ in the previous model does not seem to work well.

Nonetheless, since the Schwarz criterion suggests accepting the restricted model, it could be that either the VAR model or the previous exercise's model would be favored by the SC. The previous model had only 9 free coefficients, whereas this one has 57, and the SC tends to strongly favor smaller models.

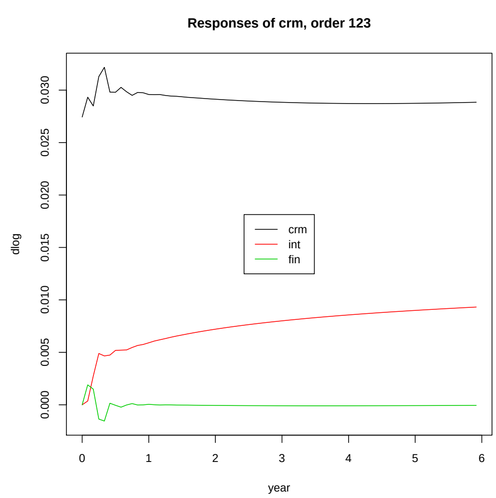
(e) Find the roots of the model — either the eigenvalues of the system matrix when it is stacked into single-lag form, or the roots of $|I - B(z)| = 0$. (They are the same.) Explain how it is possible from looking at the list of roots to determine whether the estimated model implies that an unconditional covariance matrix for the process exists.

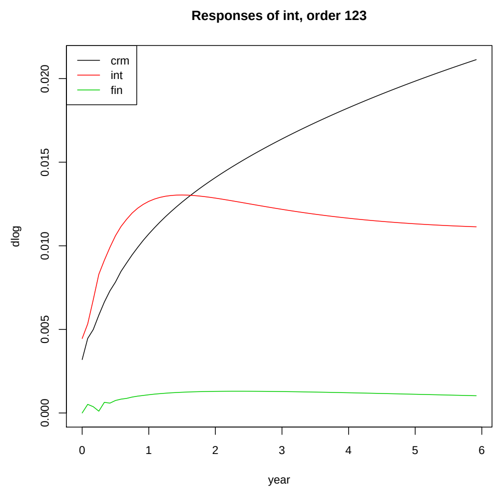
The roots are

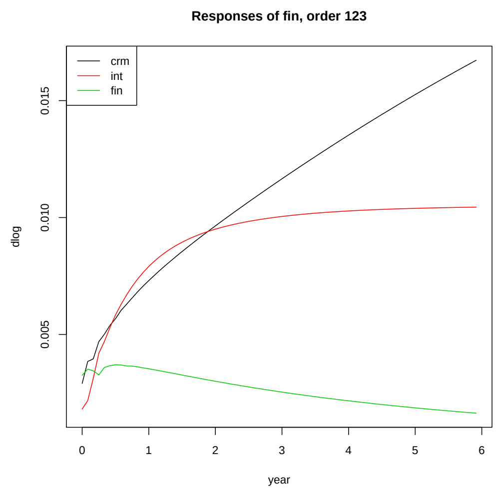
```
 [1]  1.00144888+0.0000000i  0.98787156+0.0000000i  0.96034549+0.0000000i
 [4]  0.85441845+0.0000000i -0.18116991+0.6707716i -0.18116991-0.6707716i
 [7]  0.62917765+0.0000000i -0.60213707+0.0000000i -0.50666609+0.2709784i
[10] -0.50666609-0.2709784i -0.29101853+0.4949737i -0.29101853-0.4949737i
[13]  0.47345538+0.3002826i  0.47345538-0.3002826i  0.15441690+0.5217151i
[16]  0.15441690-0.5217151i  0.18605863+0.0000000i -0.07704899+0.0000000i
```

Since one is larger than one, there is no stationary distribution. Note that the third is close to one, but in monthly data implies a half-life (time to decay to 50% of original size) of 17 months, which is not long relative to the length of business cycles. The second root has a half-life (log(.5)/log(root)) of 57 months, nearly five years. This is still small relative to the sample size of 731 months, so treating it as precisely one might distort the model dynamics.

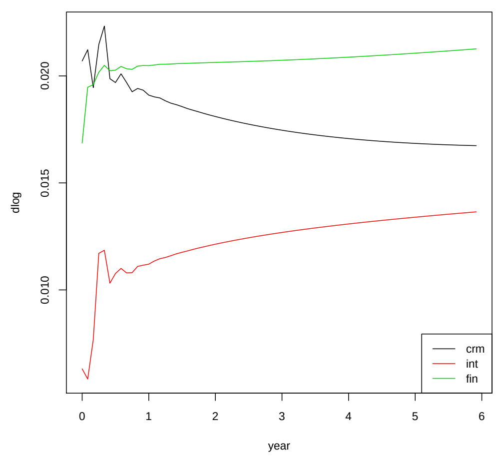
The model's impulse responses include some that increase nearly linearly over 6 years. This behavior cannot be from the single root slightly larger than one — exponential growth at that rate would be very slow. It must be from some of these large roots concatenating to produce near-repeated-root behavior.



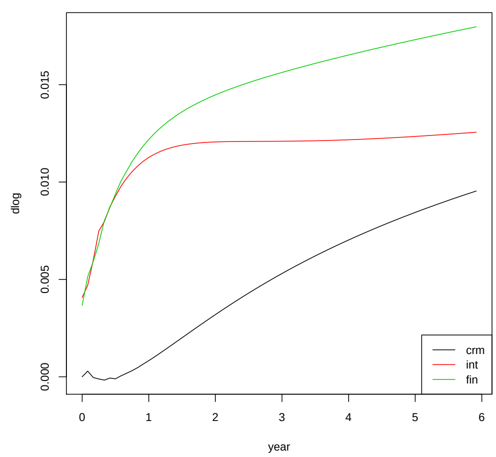

Responses of int, order 123
crm
int
fin
dlog
0.000
0.005
0.010
0.015
0.020
0
1
2
3
4
5
6
year

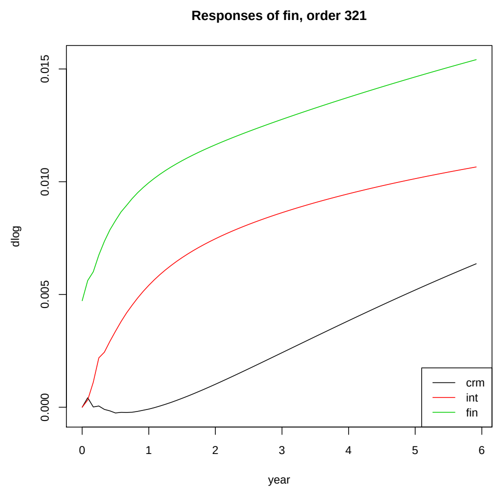

Responses of fin, order 123
crm
int
fin
dlog
0.005
0.010
0.015
0
1
2
3
4
5
6
year

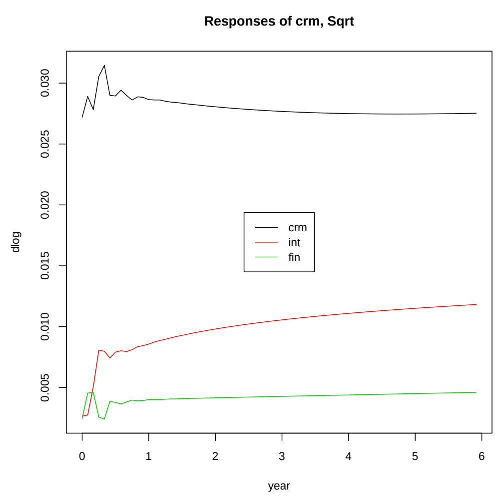
Responses of crm, order 321

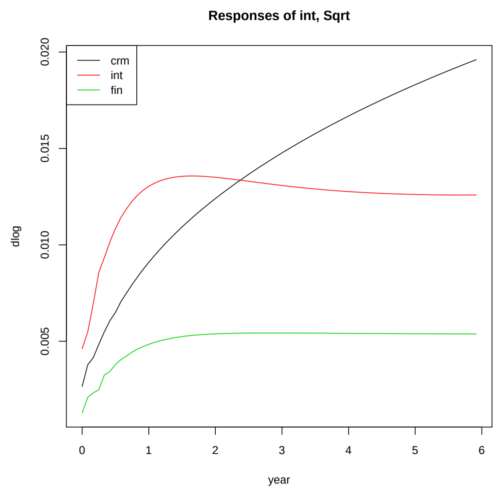


Responses of int, order 321




Responses of fin, order 321
dlog
0.000
0.005
0.010
0.015
0
1
2
3
4
5
6
year
crm
int
fin


Responses of crm, Sqrt
dlog
0.005
0.010
0.015
0.020
0.025
0.030
0
1
2
3
4
5
6
year
crm
int
fin


Responses of int, Sqrt
crm
int
fin
dlog
0.005
0.010
0.015
0.020
year
0
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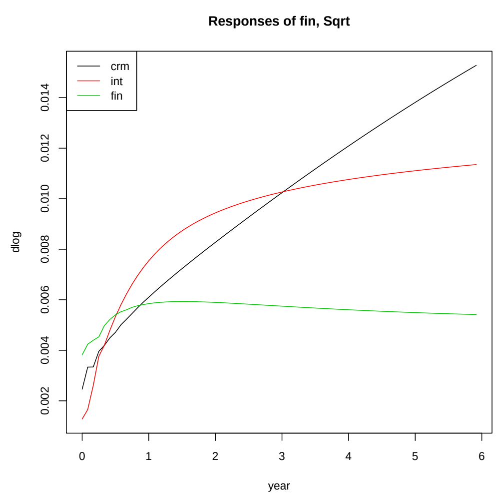
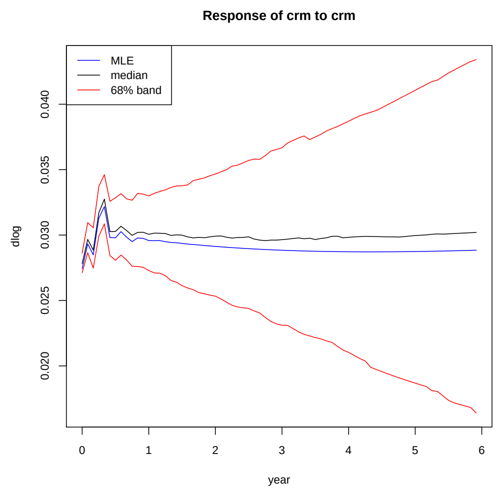
6


Responses of fin, Sqrt
crm
int
fin
dlog
0.002
0.004
0.006
0.008
0.010
0.012
0.014
year
0
1
2
3
4
5
6


Response of crm to crm
MLE
median
68% band
dlog
0.020
0.025
0.030
0.035
0.040
0
1
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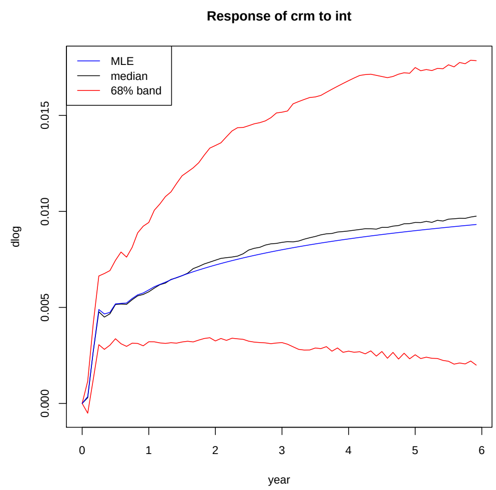
6
year


Response of crm to int
MLE
median
68% band
dlog
0.000
0.005
0.010
0.015
0
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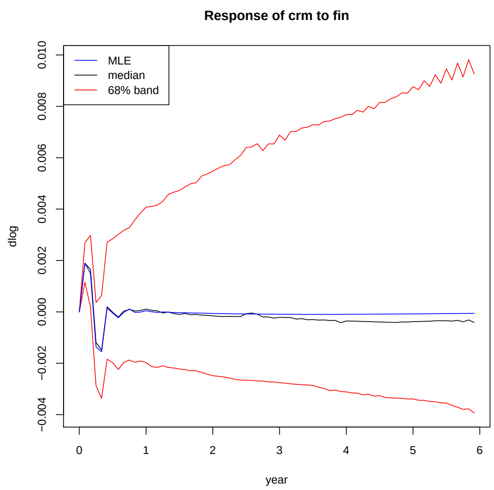
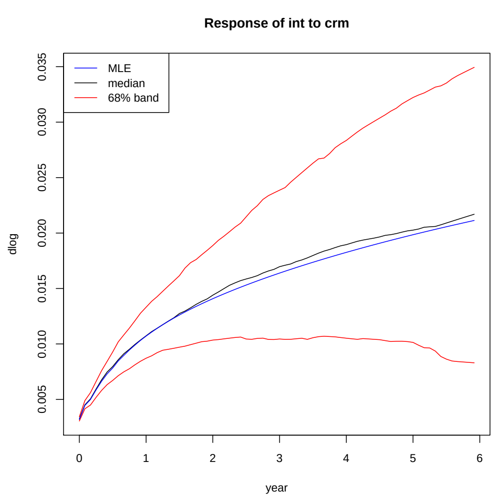
6
year


Response of crm to fin
MLE
median
68% band
dlog
0.010
0.008
0.006
0.004
0.002
0.000
−0.002
−0.004
0
1
2
3
4
5
6
year


Response of int to crm
MLE
median
68% band
dlog
0.035
0.030
0.025
0.020
0.015
0.010
0.005
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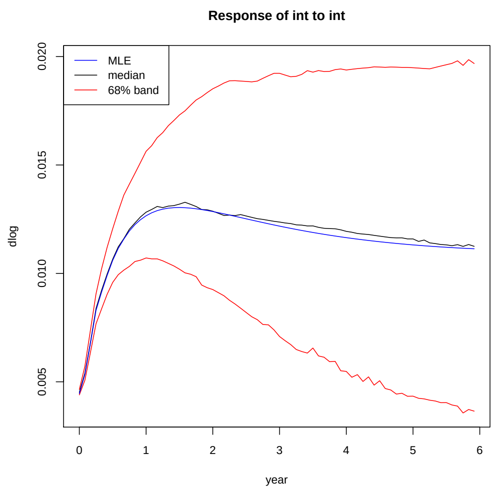
year


Response of int to int
MLE
median
68% band
dlog
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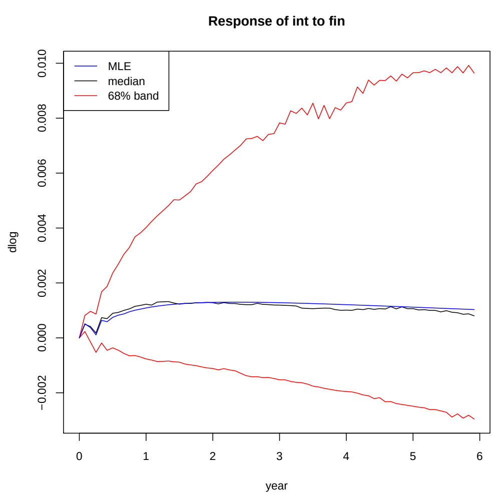
Response of int to fin
MLE
median
68% band
dlog
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0.002
0.004
0.006
0.008
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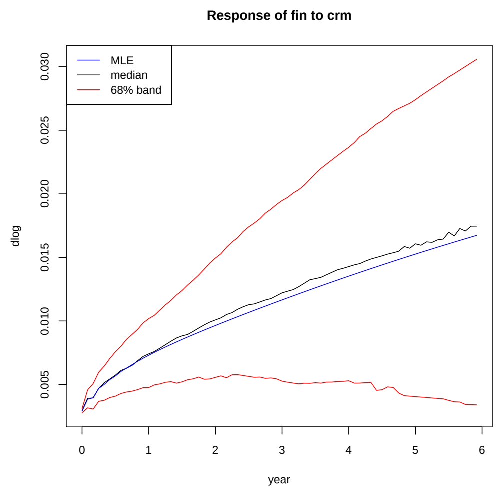
year


Response of fin to crm
MLE
median
68% band
dlog
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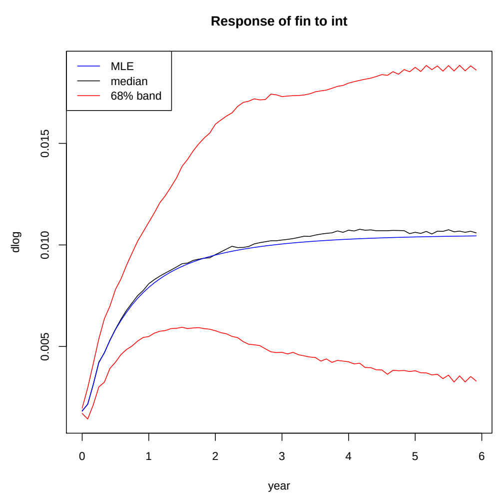
Response of fin to int
MLE
median
68% band
dlog
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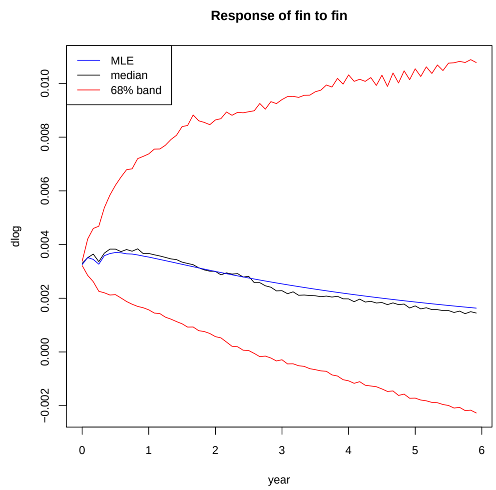
Response of fin to fin
MLE
median
68% band
dlog
year
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