MCMC EXERCISE

(1) (a) Devise a MCMC sampling scheme to generate a sample from a pdf for (a, b) that is proportional to

$$p(a,b) = ((a-1)^2 + (b-1)^2)e^{-\sqrt{(a-1)^2+(b-1)^2)}-.1a} .$$

(b) Use the scheme to generate a sample whose "effective size", for both a and b, exceeds 200. Also find the effective sample size for $(a-1)^2 + (b-1)^2$.

(c) Redo the effective sample size calculation using a "thinned" sample consisting of every nth observation, where n is big enough to reduce the sample to around 1000 observations.

(d) Display trace plots and density plots for the sampled a, b, and $\arctan(b/a)$. These can be based on the thinned sample if that has effective sample size roughly similar to the effective sample size of the unthinned sample. (The R `coda` package, once installed, makes `plot(mcmc(draws))` produce a set of trace plots and density plots for the N MCMC draws of k parameters in the $N \times k$ matrix `draws`. There may be something similar for the Matlab version of the coda package.)

(e) Make a contour plot of the p function over the range $-6, 8$ for a and b.

(f) Make a scatter plot of the draws, using dots to represent the points. (In *R*, if `draws` is a two-column matrix containing the draws of a, b pairs, you would use `plot(draws, pch=".")` to accomplish this.)

(g) The density

$$q(a,b) = \frac{4}{3\pi}(a^2+b^2)e^{-2\sqrt{a^2+b^2}}$$

is proper, i.e. integrates to one. It looks like it should work as the weight function in a Modified Harmonic Mean calculation of the marginal data density on this model, if recentered at (1,1). Explain why it needs to be recentered and why its tail behavior looks ok for this purpose.

(h) Use the modified harmonic mean method with this weight function to evaluate the integral of the density function kernel p. Is effective sample size for q/p as good as for a and b? Is the standard deviation of q/p small enough to give you confidence in your estimate of the scaling constant? [Unfortunately, there is not much interesting to be done with a scaling constant estimate except when models are being compared, which we are not doing here.]

Date: November 13, 2008.