ANSWERS TO ARMA FIT EXERCISE

The optimized estimates for the three models are shown below. As was mentioned in class, the exponential prior on $1/\sigma^2$ turned out to imply almost no probability on values of σ^2 as low as the sample forecast variance that can easily be obtained even with very simple models. So the columns corresponding to that prior has unrealistically high estimated residual variance. It is also only in these columns that there is a substantial moving average coefficient. All the models show one estimated root very close to one. The modal log posterior values are possibly not comparable across models. Because of the use of the "factor proportional to a $N(0, \sigma^2)$ pdf for $\bar{y} \cdot (\sum \rho_i - 1) - \alpha$", the prior has a normalizing factor that is not accounted for in these results and might differ somewhat across models with different numbers of AR lags. It is nonetheless clear that the models with the $1/\sigma^2$ prior are dominated in fit by the ones with the exponential prior, as might be expected. The models with the exponential prior are all within a range of 2.5 natural log units in maximum posterior pdf value, which means that there is little difference between them in fit. Differences of this magnitude might easily disappear if the normalizing constant is accounted for properly or if the prior were to change slightly, say by using different variances on the lagged ρ's.

Results for ARMA(5,1)

	exp'l prior on $1/\sigma^2$	exp'l prior on σ^2
ρ_1	0.9730576	1.378733653
ρ_2	0.0615118	-0.304253967
ρ_3	-0.0335163	-0.153651609
ρ_4	-0.0101222	-0.001016871
ρ_5	0.0080348	0.079394094
α	0.0167614	0.012402301
θ	0.1650601	-0.088899098
σ^2	0.0091286	0.000075938
$\sum\rho$	0.998986	0.9992053
log posterior pdf	-412.362	-923.163

Date: November 23, 2006.

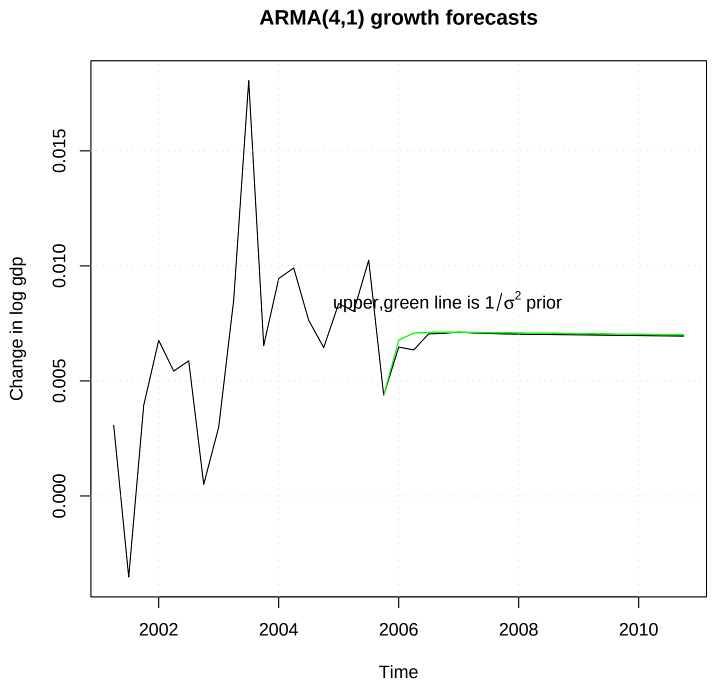
Results for AR(5)

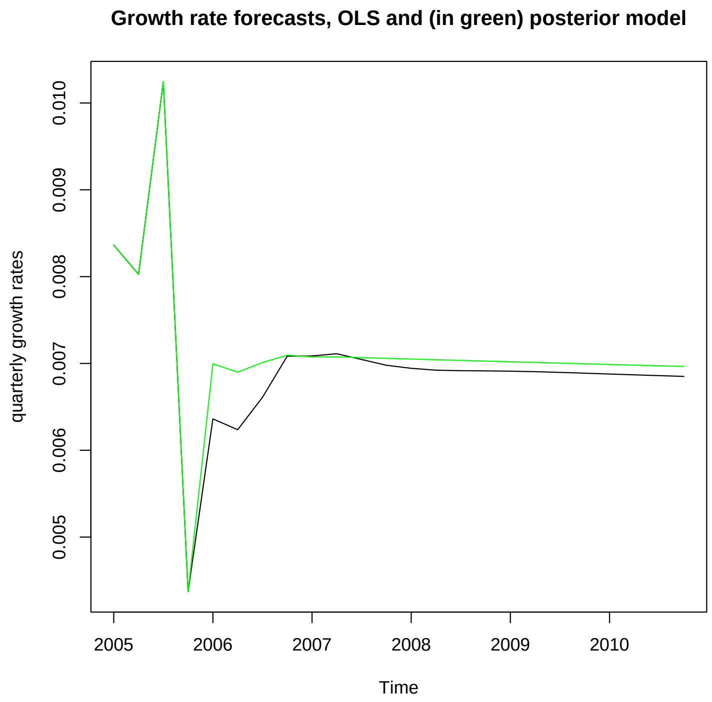
	exp′l prior on $1/\sigma^2$	OLS	exp′l prior on σ^2
ρ_1	1.12085676	1.29586028	1.29391029
ρ_2	-0.04892733	-0.19771312	-0.19399753
ρ_3	-0.06653617	-0.17004820	-0.16854483
ρ_4	-0.01791756	-0.01830424	-0.01585762
ρ_5	0.01162761	0.08922339	0.08362628
α	0.01412293	0.01451564	0.01341182
σ^2	0.00912834	0.00007625	0.00007593
$\sum \rho$	0.9991033	0.9990181	0.9991366
log LH at max			-924.4888

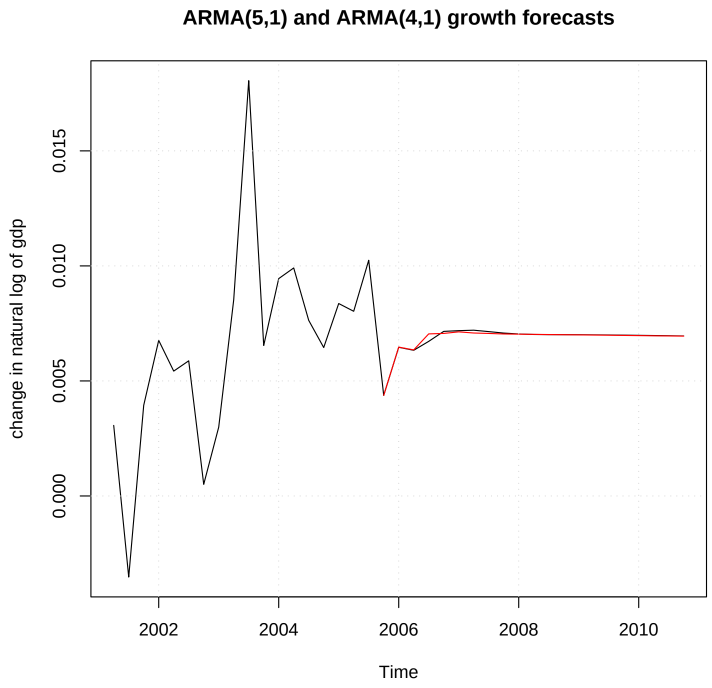
Results for ARMA(4,1)

	$\exp(-1/\sigma^2)/\sigma^4$ prior	$\exp(-\sigma^2)$ prior
ρ_1	0.97903216928921	1.348614e+00
ρ_2	0.04897836498979	-2.724487e-01
ρ_3	-0.02410658883885	-1.759705e-01
ρ_4	-0.00493252498719	9.904819e-02
α	0.01663006071991	1.182957e-02
θ	0.16119425533512	-4.926028e-02
σ^2	0.00912885823666	7.650896e-05
log posterior	-411.0938	-921.9466

Forecasts with the AR5 and ARMA(4,1) and ARMA(5,1) models are shown in the plots below. All of the forecasts revert rather quickly to a steady state growth rate of 2.8% at an annual rate and follow similar paths.


ARMA(4,1) growth forecasts
Change in log gdp
0.000
0.005
0.010
0.015
upper,green line is $1/\sigma^2$ prior
2002
2004
2006
2008
2010
Time


Growth rate forecasts, OLS and (in green) posterior model
quarterly growth rates
0.005
0.006
0.007
0.008
0.009
0.010
2005
2006
2007
2008
2009
2010
Time


ARMA(5,1) and ARMA(4,1) growth forecasts
change in natural log of gdp
0.000
0.005
0.010
0.015
2002
2004
2006
2008
2010
Time