VAR EXERCISE

The course web site has data on the federal funds rate (ffr), the M1 money stock (m1), the price level (cpi), and industrial production (ip), both as an **R** time series object in `rmpy.R` and as a formatted text file `rmpy.txt`. The data are monthly, 1960:1 to 2005:3. Use these data to estimate a reduced form VAR. Use the default values for the sum of coefficients and cointegration dummy observations in `rfvar3.R` or `rfvar3.m` to do the estimation. If you use other software, you will need to figure out for yourself how to implement the corresponding dummy observations. Log the values of the non-interest-rate variables before you estimate, and it can ease interpretation also to devide the ffr series by 100, so all the residuals are expected to be the same order of magnitude.

(a) Find the eigenvalues of the estimated system. Do they separate cleanly into near-unit roots and others? Do the eigenvectors show evidence of near-repeated-root behavior? [Hint: My own calculations using the default dummy observation prior, with not quite the same dataset, showed four or more roots very close to one, plus clear evidence of repeated-root-like behavior.]

(b) From your estimates, form the sum-of-coefficients matrix that has reduced rank and determines cointegrating vectors in a VECM model. Does the eigenvector-eigenvalue decomposition of this matrix look close to the form expected when there is cointegration?

(c) Calculate forecasts of the four variables from the beginning-of-sample initial conditions. (Here `fcast.R` or `fcast.m` may be useful.) Plot the actual and forecast values for each series, and on each plot show also the unconditional mean (if it exists) of the variable. (The mean will only exist, of course, if your estimates have all roots at least slightly less than one in absolute values.).

(d) Compare all the results you have obtained above with what emerges when you estimate the VAR with no prior — i.e. with $\lambda = \mu = 0$ in the arguments to `rfvar3`.

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Checking for near-repeated-root behavior. Whenever a matrix A has eigenvalues close to each other that correspond to eigenvectors that are also close to each other, it can generate impulse responses that, for small t, behave like $\lambda^t Q(t)$, where $Q(t)$ is a polynomial in t and λ is the value to which both roots are close. When the roots are close to one, this implies the impulse responses can behave like polynomials in t over long stretches of time. Suppose x and $y = x + z$ are eigenvectors that are nearly the same corresponding to two roots λ and $\lambda - \nu$ that are also nearly the same. Then $A^t \cdot (x - y) = \lambda^t x - (\lambda + \nu)^t (x + z)$. To first-order accuracy, this is $\lambda^t z + t\lambda^{t-1} \nu x$. When $\lambda = 1$, or close to it, the impulse response is an initial small perturbation z, plus a linearly growing vector of the from tx. When there are more than two roots with similar roots and eigenvalues, there are linear combinations of the eigenvectors that produce close approximations to higher-order polynomial behavior.

To check for this kind of behavior, one first checks whether there are collections of very similar eigenvalues. If so, one can check the matrix of eigenvectors for near-singularity, by calculating either an eigenvalue decomposition or a singular value decompsition of that matrix. Of course near-singularity of the eigenvector matrix does not imply directly that there are similar eigenvectors corresponding to similar roots; there coule be similar eigenvectors corresponding to dissimilar roots. So at this point one might try to use eyeball methods to see if there are similar eigenvectors aligned with similar roots. Note, though, that any time eigenvectors are similar, the implication is that small perturbations (the difference of the two eigenvectors) can produce responses that for a while grow bigger.

Similar roots combined with similar eigenvectors close to one are particularly interesting, because they imply that impulse responses to some patterns of disturbance can grow steadily for a long time, even though there are no roots bigger than one.