EXERCISE DUE MONDAY, 10/24

(1) CPINS is 19 in 1921, almost 200 in 2005. So the size of rounding effects themselves increases by a factor of 10 over the sample. The standard deviation of inflation calculated as specified in the exercise (growth in logs at annual rate) is 9.6 over the 1921-53 sample, 4.0 over the 1953-2005 sample, i.e. they differ by a factor of over 2.4. If we treated the ratio of variances as an F statistic, the F would be around 5.8 and highly significant, given the large numbers of degrees of freedom. The fact that the inflation numbers are not serially independent and not at all normally distributed makes the F a very inexact approximation here but it seems clear there are differences in variance. The plot in Figure 1 shows clearly the decline. Rounding error is visible in the obvious "up-down, up-down" patterns in the data. These occur because after the first move from, say, 19.1 to 19.2, the 19.2 figure is likely

Date: October 31, 2005.

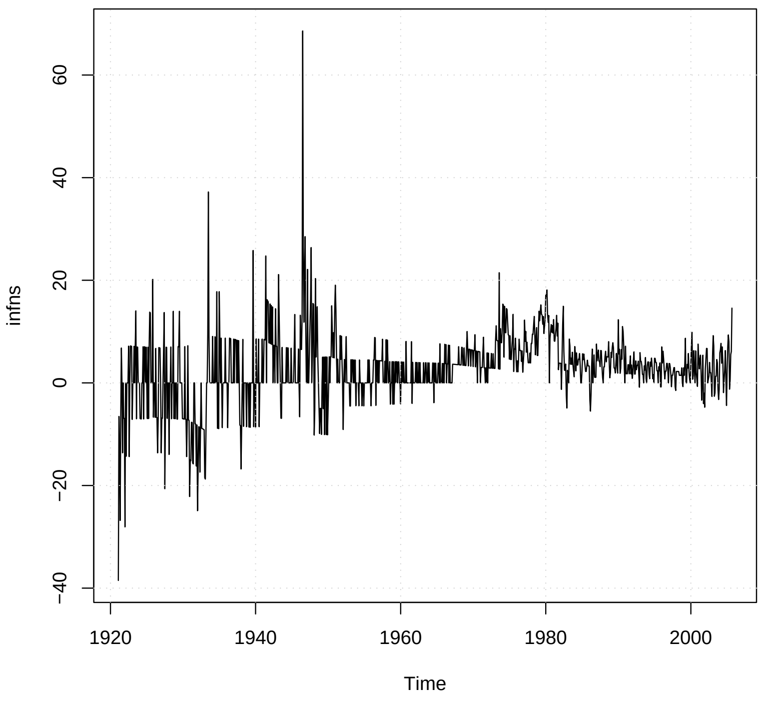

FIGURE 1. CPI Inflation, NSA

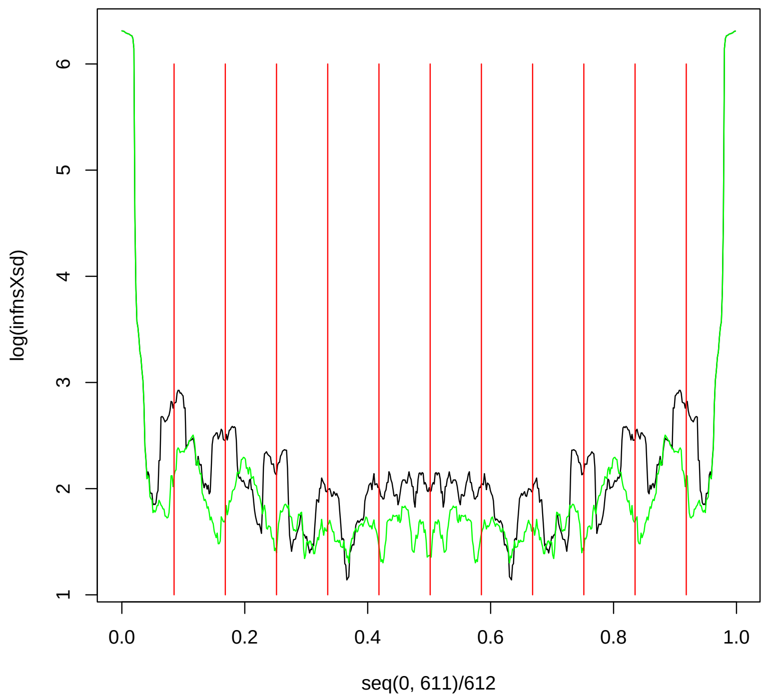

FIGURE 2. Spectral densities of inflation: SA green, NSA black

to have been rounded up. Therefore it is less likely than normal, for a period or two, that it will increase again. If inflation tends to move smoothly, this will generate a pattern of oscillation between non-zero and zero inflation rates. It is easy to see how big these up-down patterns are, from the graph, and they do indeed change by a factor of 10, becoming almost invisible at the end of the sample. But even at the beginning, their contribution cannot have a standard error of more than about one half percentage points ($100 \times .1/20$), while the actual standard deviation of inflation before 1950 is many times that large, as is the decline. So while this is a potential source of bias worth considering, it does not seem here to be big enough to make us doubt that there is an actual decline in inflation variance.

I've calculated all spectral densities and cross-spectral densiities, for convenience, for the dates September 1954 through August 2005 (so all three series cover all dates and there are an even number of full years). Figure 2 shows spectral densities for the adjusted and unadjusted inflation data. The unadjusted spectral density has clear, though not very big compared to many other macro series, seasonal peaks at 12,6,4,and 3 month cycles. The

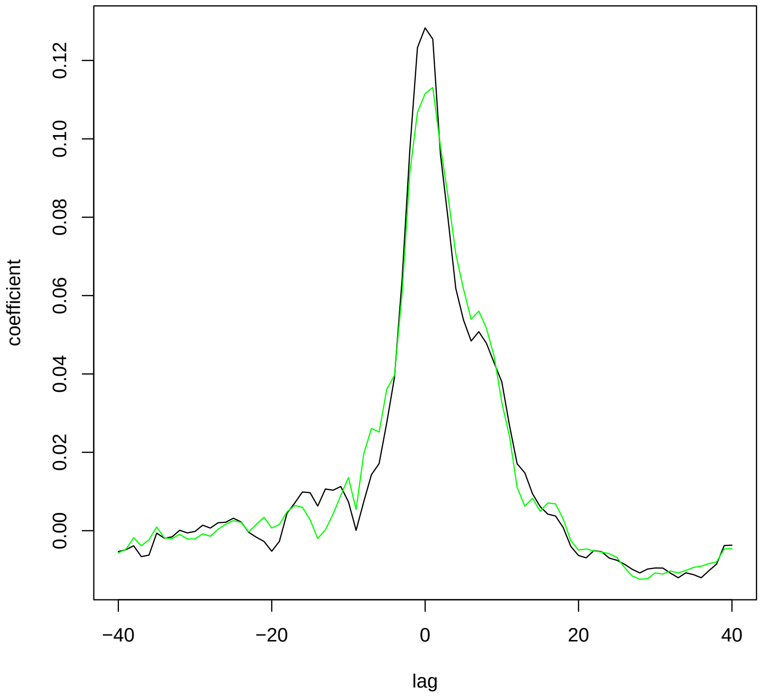

FIGURE 3. Lag distributions for FFR projected on inflation: NSA black, SA green

2.4 and 2 month frequencies do not stand out from the rest of a broad range of elevated power at high frequencies that may reflect the rounding error.

The adjusted series seems to have peaks just above the first two seasonal frequencies. This is what would be expected from creating dips at seasonal frequencies — there then have to be peaks in between. The adjusted series also has much reduced power at *all* high frequencies, not just at the seaasonals. The two spectral densities are essentially identical at frequencies longer than a year, as they should be.

Figure 3 shows the result of projecting FFR on inflation, using the frequency domain. The fft's were done with 612 elements, and these plots show elements 573 to 612, followed by 1 to 41, of the inverse transform. The plots are similar, with a smooth peak and slightly more weight on the past (which is the positive lags, to the right) than on the future. There is little evidence of seasonal distortion, but the lag distribution based on the adjusted data is more sharply peaked. Because the right-hand-side variable has high frequency variation smoothed out, the lag distribution can do less smoothing.

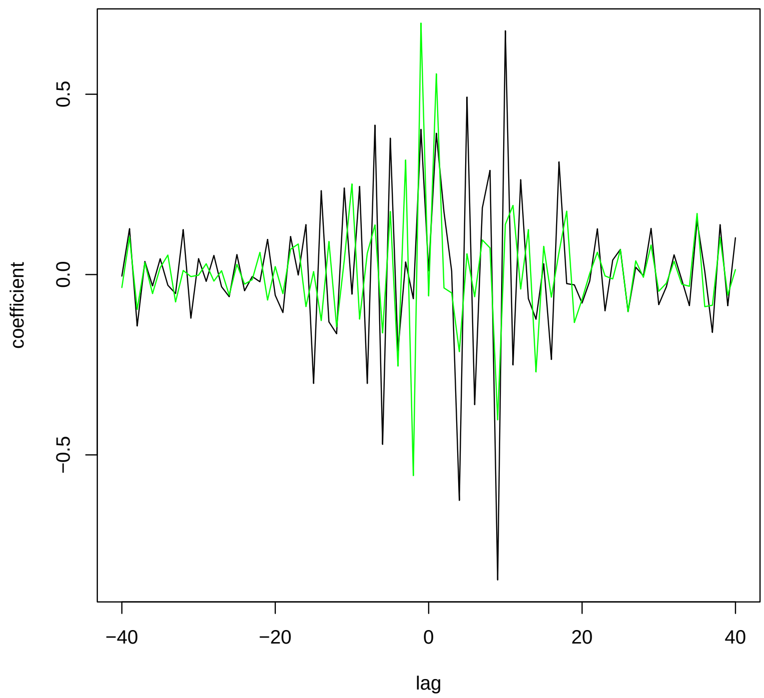

FIGURE 4. Lag distributions for inflation projected on FFR: NSA black, SA green

With the funds rate on the right-hand side, we get Figure 4. Now the lag distribution has to "unsmooth" the smooth right-hand side variable, and the differences between adjusted and unadjusted data are perhaps more noticeable. With the unadjusted data, more of the large weights are on the past, whereas if anything the opposite is true for the adjusted data. This could distort conclusions. [Second year paper idea, in case you haven't started one: Examine the sensitivity of conclusions in papers in the literature about whether the Phillips curve and/or the monetary policy reaction function are "forward-looking" to the use of seasonally adjusted data.]

Footnote: I did the displayed calculations with the 0-frequency fft's set to zero. I did not bother to set the fft's to zero at the exact seasonal harmonics in those displayed. Because of the weak seasonality, results are little affected by taking out seasonals (or even the zero frequency, for that matter).

(2) *Construct an ARMA operator of the form $P(L)/Q(L)$, with P second order and Q of order 2 or three, both one-sided and invertible, and with the property that its Fourier transform, $P(e^{-i\omega})/Q(e^{-i\omega})$ has a single large spike at $\pi/2$. Hint: You may want to give numerator and denominator both complex roots, near one in*

absolute value and near each other, but not equal. After you have constructed your ARMA operator, plot a simulated draw of 100 observations from a process with your operator as its MA operator. You can do this several ways. One approach is to draw i.i.d. shocks and then simulate $Q(L)y_t = P(L)\varepsilon_t$ forward in time. Another is to calculate the spectral density, inverse FT it to get the acf R_X, populate a covariance matrix with R_X values, Cholesky factor it, and multiply an i.i.d. normal vector by that factor.

I should have been more specific. You can get a spike of sorts any time you make $Q(L)$ have a root at $\rho e^{i\theta}$, where ρ is slightly larger than one and $\theta = \pi/2$. The trick, which I didn't ask you for explicitly, is to get this spike to be narrow and to make $P(e^{-i\omega})/Q(e^{-i\omega})$ nearly one away from the peak. That way, it can nearly exactly undo a seasonal adjustment filter that operates mainly at $\pi/2$. A way to do this is to make $P(z)$ second-order with pair of complex roots at $.98^{-1}e^{\pm i\pi/2}$, while $Q(z)$ is also second order, but with complex roots at $.99^{-1}e^{\pm i\pi/2}$. The ratio of these polynomials is nearly 1 for z on the unit circle, except near $z = e^{\pm\pi/2}$. To be specific, this makes the operator

$$\frac{1 - .9604L^2}{1 - .9802L^2}\,.$$

More generally, we could make the root be at $\rho^{-1}e^{\pm 2\pi j/12}$ by using $1 - 2\rho\cos(2\pi j/12) + \rho^2 L^2$ as the polynomial, or as a factor in the polynomial.

Plots of the real and imaginary parts of $P(e^{-i\omega})/Q(e^{-i\omega})$ are shown in Figure 2. Note that the real part is near one and the imaginary part near zero except in narrow bands near $\pi/2$ and $3\pi/2$.

(3) *The Hodrick-Prescott filter applied to a series X_t, $t = 1, \dots, T$ delivers a filtered "trend" estimate $\hat{X}_t$ that minimizes*

$$\sum_{t=1}^{T}(X_t - \hat{X}_t)^2 + \lambda \sum_{t=3}^{T}(\hat{X}_t - 2\hat{X}_{t-1} + \hat{X}_{t-2})^2\,.$$

As λ goes to ∞, $\hat{X}$ converges to a straight line, and as $\lambda \to 0$ it converges to the X_t sequence itself. If we are not close to the end points of the sample (i.e., to t of 1 or T), the first-order conditions of this minimization problem imply

$$(1 + 6\lambda - 4\lambda(L + L^{-1}) + \lambda(L^2 + L^{-2})\hat{X}_t = X_t\,.$$

Calculate the FT of the Hodrick-Prescott filter for various values of the weight λ on squared second differences. Do this by ignoring endpoints, so that the filter is characterized by $\hat{X}_t = B(L)X_t$, where $B(L)$ is the inverse of the polynomial in L that appears on the left-hand side above. Determine what values, if any, of λ keep the filter's FT close to one for frequencies between zero and 6 years, close to zero outside that range, for quarterly and for monthly data. (Two different values of λ, if any.) You can do this with the following R code:

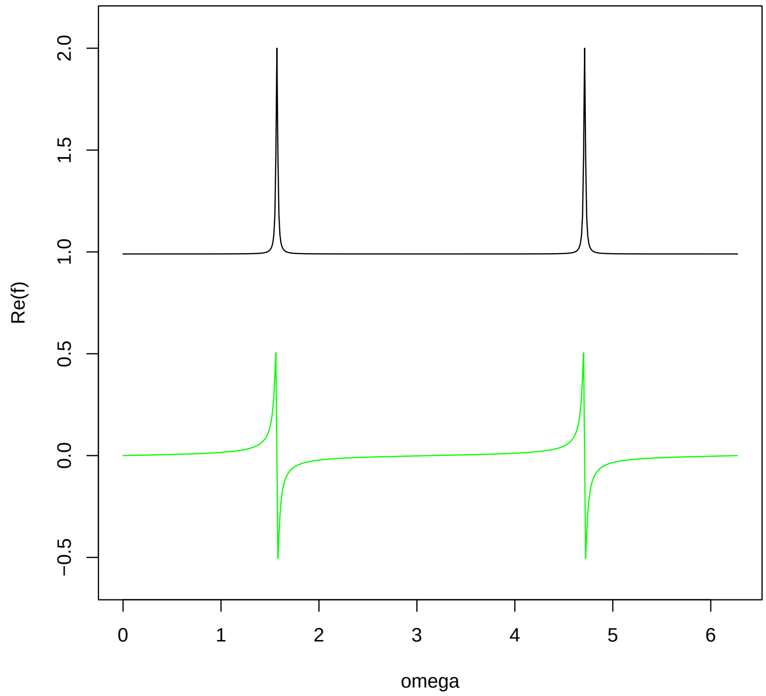

FIGURE 5. Real (upper) and imaginary (lower) parts of P/Q

```
hpt <- function(lambda){
omega <- 2*pi*(0:599)/600 ft <- 1/(1 + lambda * (6 - 8 *
cos(omega) + 2 * cos(2*omega)))
plot(omega,ft,type="l")
return(ft)
}
```

You'd want to preserve the returned FT'd filters until you'd found one that concentrates in the desired bands. It looked to me as if $\lambda = 2000$ works fairly well for quarterly data and $\lambda = 100000$ works fairly well for quarterly. Figure 6 shows the FT'd filter and the 6-year frequency ($2\pi/72$ for monthly, $2\pi/24$ for quarterly) for these $\lambda = 2000$ and $\lambda = 100000$ cases. I think the usual practice is to use $\lambda = 1600$ for quarterly data.

The final graph here was done with

```
plot(omega[1:60],ft2000[1:60],type="l",col="green")
lines(2*pi*c(1,1)/24,c(0,1),col="red")
lines(omega[1:60],ft100K[1:60],type="l")
lines(2*pi*c(1,1)/72,c(0,1),col="red")
dev.copy2eps(file="hp2K100K")
```

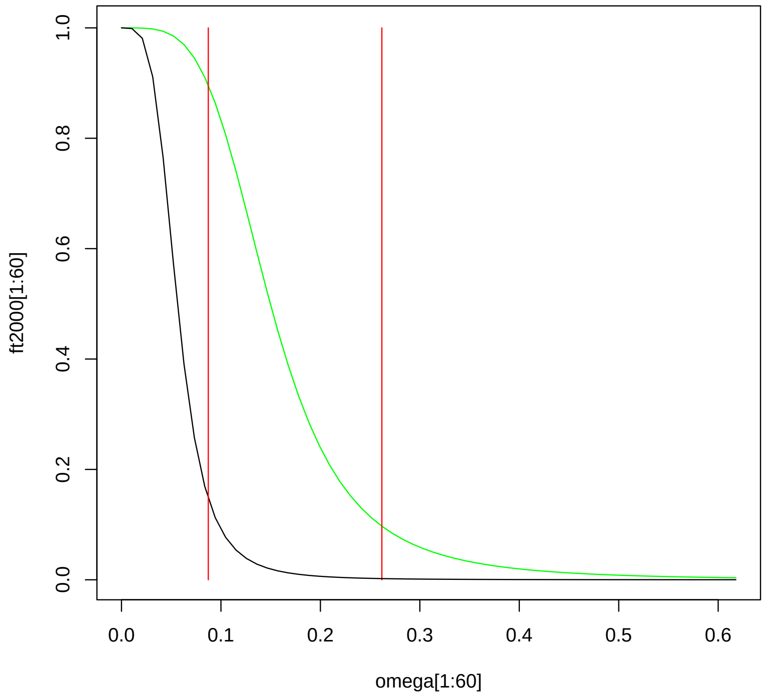

FIGURE 6. HP filter in the frequency domain: Quarterly and Monthly, with 6-year frequency marked