MID-TERM EXAM

There are 5 questions, including one on a second page. Answer all 5 questions. There are 90 points in total, including 5 bonus points awarded free to everyone who hands in the exam.

(1) (15 points) Suppose the Gaussian process X_t satisfies

$$\mathrm{Cov}(X_t, X_{t-s}) = \begin{cases} 1 & s \text{ even} \\ 0 & s \text{ odd} \end{cases} .$$

(a) Is this process stationary?
(b) Is this process linearly regular? If so, display its moving average representation.
(c) Is this process linearly deterministic? If so, determine $E[X_t \mid \{X_{t-v},\, v \geq 99\}]$.
(d) Is this process ergodic? Prove your answer is correct.

(2) (15 points) Consider the function

$$S_X(\omega) = e^{-(\pi-\omega)^{-1}(\pi+\omega)^{-1}}, \quad \omega \in [-\pi, \pi] .$$

(a) Sketch the shape of this spectral density function. Is it the spectral density of a stationary Gaussian process? How do you know?
(b) Is it the spectral density of a linearly regular process? How do you know?

(3) (15 points) $y_t = \varepsilon_t + .7\varepsilon_{t-1} - .4\varepsilon_{t-2}$, where ε process is i.i.d. $N(0,1)$.
(a) What is the variance of y_t conditional on knowledge of $\{\varepsilon_s,\, s < t\}$?
(b) What is the variance of y_t conditional on knowledge of $\{y_s,\, s < t\}$?

(4) (25 points) y_t's fundamental MA representation is $y_t = \varepsilon_t + \alpha\varepsilon_{t-1}$.
(a) Write out the plant and observation equations for a Kalman filter, treating $s_t = (\varepsilon_t, \varepsilon_{t-1})$ as the state and α as known.
(b) With $y_1 = 1$, $y_2 = 0$ being the whole sample, calculate filtered and smoothed estimates of the state at $t = 1, 2$, assuming $\alpha = .7$ and that the initial prior covariance matrix for s_1 is the identity. Also calculate the log marginal data density (likelihood times prior) for this value of α. 2-decimal place accuracy is enough, and for the likelihood you can leave logs unevaluated, if you're not doing this on a calculator.

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(5) (15 points) Though there is no unique optimum deseasonalizing filter, such a filter ought to remove power in a narrow band about each seasonal frequency and change the series it is applied to as little as possible at non-seasonal frequencies. With these qualities in mind, find the Fourier transforms of each of the filters below and discuss how well each one meets the criteria for a deseasonalizing filter for monthly data.

(a) $\frac{1-L^{12}}{1-L} = \sum_{s=0}^{11} L^s$

(b) $1 - \sum_{s=-3}^{3} L^{12s}/7$

(c) $1 - \sum_{s=-3}^{3} L^{4s}/7$

(d) $1 - \sum_{s=0}^{3} L^{12s}/4$