EXERCISE DUE MONDAY, 10/3

(1) The probability space is $S = \{1,2,3,4,5\}$. The probability of every point ω in S is $1/5$. We define random variables X_i by

$$\begin{aligned} X_1(5) &= 2 & & \\ X_1(\omega) &= 1 & & \omega < 5 \\ X_2(1) &= 2 & & \\ X_2(\omega) &= 1 & & \omega > 1 \\ X_3(3) &= 2 & & \\ X_3(\omega) &= 1 & & \omega \neq 3\,. \end{aligned}$$

Let $\mathcal{F}_t$, $t = 1,\ldots,3$ be defined as the σ-field generated by X_s, $s < t$.

(a) Display the sets making up each of $\mathcal{F}_1$, $\mathcal{F}_2$ and $\mathcal{F}_3$.
(b) Could these three random variables form part of a stationary process?
(c) Find $\mathrm{Cov}(X_i, X_j)$ for all combinations of $i, j = 1,\ldots,3$.
(d) Find $E_t[X_3]$ and $\mathrm{Var}_t[X_3]$ for $t = 1,2$, evaluated at $X_1 = 1, X_2 = 1$, at $X_1 = 2, X_2 = 2$, and at $X_1 = 2, X_2 = 1$. Note that these variables are not joint normal; the conditional expectations will not be linear functions.

(2) (a) For each of the sets of moving average weights a below, compute and plot the acf of $X_t = \sum a_i \varepsilon_{t-i}$ for time separations $s = -15,\ldots,15$. This will be tedious unless you use the computer.
(b) For each of the sets of moving average weights a below, compute and plot 5 simulated draws for X_t, $t = 1,\ldots,50$ by generating 60 i.i.d. $N(0,1)$ random draws and averaging them with a. Note that you can draw a single set of 5 i.i.d. ε sequences and use the same 5 for each of the a's. This makes it clearer what differences are due to the a's alone. All 5 lines for a single a should be on the same plot.

The a's:

(a) $a_i = 1, i = 0,\ldots,10$
(b) $a_i = \sin(2\pi i/10) + 1, i = 0,\ldots,10$
(c) $a_i = \cos(2\pi i/10) + 1, i = 0,\ldots,10$
(d) $a_i = (-1)^i, i = 0,\ldots,10$

Command to generate a 60 by 5 matrix of $N(0,1)$ random variables:
matlab: `z = nrand(60,5)`
R: `z <- matrix(rnorm(60*5),ncol=5)`

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(3) There are, or will be, monthly data on the Federal Funds rate on the course web site. Using these data, find a maximum likelihood estimate of the weights a in a 12th-order Gaussian MA model with constant mean $\bar{r}$ for these data. Determine whether the the MA weights to which your estimates have converged are fundamental. [You can use "root-flipping", which we will probably cover in the 9/28 lecture, or you can try starting the maximization from a different place to get convergence to a different set of weights, so you can compare a_0's, or you can construct an approximation to the one-step-ahead predictor by using a large finite number of lags and see if its residual variance is close to a_0^2.]