SIMPLE MODEL COMPARISON MCMC

Here is a time series for y_t, $t = 1\dots,10$:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}.$$

You are to find the posterior probabilities on two models for this series. One model is that they are i.i.d. draws from a distribution in which $P[y_t = 1] = p$, $P[y_t = 0] = 1 - p$, with a prior on p that is uniform on $[0,1]$. The other model is that they have been generated from a Markov process in which $P[y_t = 1 \mid y_{t-1} = 1] = p_1$ and $P[y_t = 0 \mid y_{t-1} = 0] = p_2$, with a prior that is uniform over the unit square for p_1, p_2. The initial observation y_1 is under this model drawn from the marginal steady-state distribution of y_t. That is, y_1 is drawn from a distribution in which $P[y_1 = 1] = \bar{p}$ and $\bar{p}p_1 + (1-\bar{p})(1-p_2) = \bar{p}$.

(a) Find the MLE's for p and for p_1, p_2 conditional on each model.

(b) Find the posterior probabilities of the two models, assuming they have equal prior probability, by direct numerical integration, using a grid of, say, 100 points in p-space and 100×100 in (p_1, p_2)-space.

(c) Using the second-order approximation to the log likelihood at its peak, calculate an approximate posterior odds ratio for the two models.

(d) Calculate posterior odds ratios for the two models by two Monte Carlo methods:

   (i) Importance sampling, using the Gaussian approximations to form the proposal distribution.

   (ii) Metropolis-Hastings sampling, using the Gaussian approximations to form the proposal distributions.

   In each case, make at least 5000 draws and present some evidence on whether your algorithm has converged. Also calculate estimates of Monte Carlo standard errors in your estimates of p, p_1, and p_2.