EXERCISE ON MODEL CHOICE

Using the quarterly data for US real GDP supplied on the course web site along with this exercise, construct $y_t = \log(RealGDP_t)$ and calculate posterior odds for the following three models and priors:

$$y_t = y_{t-1} + \varepsilon_t \qquad \sigma^2 \sim \sigma^{-2} e^{-1/\sigma^2} \tag{a}$$

$$y_t = c + \rho y_{t-1} + \varepsilon_t \qquad \begin{cases} \sigma^2 \sim \sigma^{-2} e^{-1/\sigma^2} \\ \rho \mid \sigma^2 \sim U(-1,1) \\ c \mid \sigma^2 \sim N(0, 10^4 \sigma^2) \\ \rho \mid \sigma^2 \text{ independent of } c \mid \sigma^2 \end{cases} \tag{b}$$

$$y_t = c + \alpha y_{t-1} + \beta y_{t-2} + \varepsilon_t \qquad \begin{cases} \sigma^2 \sim \sigma^{-2} e^{-1/\sigma^2} \\ \alpha, \beta \sim \text{uniform on } |\beta| < 1,\ \alpha + \beta < 1,\ \beta - \alpha < 1 \\ c \mid \sigma^2 \sim N(0, 10^4 \sigma^2) \\ (\alpha, \beta) \mid \sigma^2 \text{ independent of } c \mid \sigma^2 \end{cases} \tag{c}$$

In each case, ε_t is taken to be $N(0, \sigma^2)$ conditional on all past values of y. So as to make things more manageable, and keep the models comparable, we assume that the first two observations on y, which we will call y_0 and y_1, are given random variables whose distributions do not depend on the model parameters. (This makes the likelihood for the model parameters depend only on the conditional pdfs implied by the three equations.) Note that for the third model, the region for α, β is exactly the set of values that imply stationary roots. Take the prior probabilities on the three models all to be one third.

In addition to the odds ratios, compute the implied $E_T[y_{T+1} \mid y_1 \ldots y_T]$, where T is the date of the last data point in the sample. Note that in principle this requires averaging across the parameter uncertainty within each model, and then across models, using the posterior pdf to supply weights. Of course in this sample it could be that one model is so much more likely than the others that no averaging is needed.

For models (b) and (c), you will need to carry out numerical integration, probably, as the non-Gaussian priors on ρ and α, β are not conjugate. In all the models, you can integrate σ^2 out analytically before proceeding to the numerical integration (if any) for the other parameters. For these one and two dimensional cases the numerical integration is straightforward. You can just choose a grid of equally spaced points on (-1,1) or over the given triangle in (α, β) space, then average the posterior over these points. Of course you will want to use the computer.