TAKEHOME FINAL EXAM
PART 2: 135 POINTS

There are four questions, numbered 2 to 5. Answer all questions, and keep your answers for these questions on separate sheets from the answers to question 1 (from part 1 of the exam).

(2) (45 points) Consider the following simple macro model:

$$\text{Taylor rule:} \qquad r_t = \alpha_0 + \alpha_1 \pi_{t-1} + \alpha_2 y_{t-1} + \varepsilon_t \tag{2.1}$$

$$\text{Phillips curve:} \qquad \pi_t = \phi_0 + \phi_1 \pi_{t-1} + \phi_2 y_{t-1} + \nu_t \tag{2.2}$$

$$\text{Fisher relation:} \qquad r_t = E_t \pi_{t+1} + \rho_0 + \xi_t\,. \tag{2.3}$$

In the Phillips curve and the Fisher relation the exogenous disturbances ν_t and ξ_t are assumed to be i.i.d. normal, with mean zero. In the Taylor rule, it is thought that the exogenous disturbance ε_t term might be serially correlated, though it is stationary and Gaussian. The three disturbances are mutually independent at all leads and lags. Their variances are unknown. The E_t operator is expectation conditioned on the values of all random variables in the system dated t and earlier.

(a) Show how to check, in the general case, whether this model implies a stationary process for r, π and y. [Hint: It may help to begin by using the Phillips curve to eliminate the expected inflation term, then reduce the system to one in π and y alone.]

(b) Check whether the system is stationary with $\phi_1 = .6$, $\phi_2 = .3$, $\alpha_1 = 1.2$, and $\alpha_2 = .2$.

(c) Can the Taylor rule coefficients be estimated consistently by applying GLS to the single Taylor rule equation? Either prove that this is always or never possible, or else give conditions on the parameter values that make it possible or impossible.

(d) Suggest a way to transform the system into one that has no serial correlation in disturbances. You will have to propose a model for the serial correlation in the Taylor rule disturbance.

(e) Are all the parameters in the equation system identified? If not, which are and which aren't?

(f) Suggest an optimal method for estimating the identified parameters of the system. Describe your method in as much detail as you can, taking account of the specific nature of this model.

(3) (45 points) It has been proposed that the US underwent a sudden slowdown its long run growth rate, perhaps in the late 70's, and also that the volatility of economic growth — the variance of its one-step-ahead forecast errors — under went a sharp reduction, probably some time in the 80's. (Though it is sometimes also proposed that in the late 90's the trend growth rate rose again, there is less consensus on this and we are going to ignore it here.)

(a) Assume the output growth rate y_t satisfies

$$y_t = \rho y_{t-1} + c_t + \sigma_t \varepsilon_t \,, \tag{3.1}$$

where $\varepsilon_t \sim N(0,1)$ is independent of $\{y_s, s < t; c_v, \sigma_v, \text{ all } v\}$. Formulate a hidden Markov chain model that captures the idea of single, randomly timed, transitions in the trend growth rate (indexed by c_t) and the volatility (indexed by σ_t). The model should imply that the timing of the two transitions is independent. Both c and σ should be modeled as constant except at their respective transition dates. The uniqueness and independence of the two transition dates can be enforced by appropriate restrictions on the form of the Markov transition matrix.

(b) Does the fact that each transition is modeled as occurring only once imply that the likelihood is ill-behaved? Why or why not?

(c) Suggest a reasonable prior for the parameters in your model and describe in detail how you would compute the peak of the posterior pdf, taking account of the specific nature of this model. Assume that the two initial observations are uninformative (i.e., condition on the initial observations).

(d) Explain how you would compare this model with one that enforces constancy of c_t and σ_t, assuming that you need posterior odds on the two models in order to guide thinking about economic policy. Give as much detail as you can about the computation, taking account of the specific nature of this model.

(4) (15 points) Discuss the following assertions, explaining why they are correct or incorrect:

(a) *The Schwarz criterion can be used to obtain consistent model selection as $T \to \infty$ when comparing models with different numbers of parameters. However in the special case where one of the models is a point, it does not work, because when there are no free parameters the Gaussian approximation to the likelihood, on which the Schwarz criterion is based, does not apply.*

(b) *When comparing regression models with dummy-observation priors, posterior odds ratios are determined entirely by the sums of squared residuals calculated by OLS across actual and dummy-observation data.*

(5) (30 points) We would like to estimate the univariate AR model with constant term

$$y_t = c + \rho y_{t-1} + \varepsilon_t \,,\; t = 1, \dots, 20\,. \tag{5.1}$$

ε_t is the innovation in y_t and is i.i.d. $N(0, \sigma^2)$, where σ^2, c, and ρ are unknown. Recognizing that use of OLS on such a model tends to give unreasonably large weight to models in which initial conditions "explain too much", we add to the actual 20 data points (21 counting the initial condition) a pair of dummy observations that append to the sample y and $X = [\mathbf{1}\; y_{-1}]$ matrices

$$\tilde{y} = \begin{bmatrix} 10 \\ 10 \end{bmatrix}\,, \quad \tilde{X} = \begin{bmatrix} 1 & 10 \\ 0 & 10 \end{bmatrix}\,. \tag{5.2}$$

(a) If the moments of the actual data are

$$X'X = \begin{bmatrix} 20 & 235 \\ 235 & 3000 \end{bmatrix}\,, \quad \sum_{t=1}^{20} y_t y_{t-1} = 3035$$

$$\sum_{t=1}^{20} y_t^2 = 3100\,, \quad \sum_{t=1}^{20} y_t = 240\,,$$

calculate the OLS estimators with and without the dummy observations.

(b) What prior pdf would give the same shape to the posterior as would be obtained by treating the likelihood for the expanded data set as if it were the posterior? Is this prior pdf a proper prior over σ^2, ρ, c? Explain your answer.