Economics 513
Fall 2002
Final Examination
Part 1: 45 points

Write your name in the space provided below

Name: ____

Please answer all questions on the exam sheet. If you need additional space, feel free to use the back of the exam pages or attach additional sheets. Each question on this part of the exam will receive equal weight.

1. Consider the Gaussian AR(1) model

$$y_t = \rho y_{t-1} + \varepsilon_t$$

where ε_t is i.i.d. N(0,1) and $y_0 = 0$.

(a) You have a random draw with T observations from this process and you are interested in the null hypotheses H_o: $\rho = \rho_o$ versus the alternative H_a: $\rho = \rho_a$, where $\rho_a < \rho_o$. Construct the most powerful test.

(b) Is this test the uniformly most powerful test for H_a: $\rho < \rho_o$? Explain. Is this test the asymptotically UMP test?

(c) Suppose that you did not know the value of y_0. What should you do? (Does your answer depend on the value of ρ_o?)

(d) Suppose that you knew the values of neither y_0 nor ρ. Construct the MLE of y_0 and ρ.

(e) Is the MLE of y_0 consistent? Explain.

(g) Derive a large sample approximation for the asymptotic distribution of the MLE of y_0. (You may assume that $0 < |\rho| \leq 1$.)