Econ. 513, Time Series Econometrics Fall 2001 Chris Sims

Confidence intervals and post-sample probability intervals*

> In addition to the exercise below, which you are to turn in for grading, you should try the exercises titled *Bayesian Basics* from the Fall 1999 version of the course. The first problem in that set is probably more algebra than is worthwhile if you try to solve it without the answer sheet, but the others are problems you should be able to solve. There is an answer sheet for this old problem set.

Consider the nonlinear regression model we discussed in class:

$$y_t = x_t^\theta + \varepsilon_t,\ t = 1, 2, 3\ . \tag{1}$$

As in class, we assume it is known that $\{\varepsilon \mid x\} \sim N(0, I)$, where the x and ε without a subscript stands for the 3×1 vectors obtained by stacking up, e.g., x_t, $t = 1, 2, 3$. We modify the discussion in class by supposing that θ can range over the entire real line $(-\infty, \infty)$. We also change the hypothetical data, so that now the observations are

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix},\ y = \begin{bmatrix} -.8 \\ 3.8 \\ 1.2 \end{bmatrix}\ . \tag{2}$$

(a) Plot the sum of squared residuals $u_t = y_t - x_t^\theta$ as a function of θ over the range (-5,2).
(b) Calculate the 95% classical confidence interval for θ based on the $\chi^2(3)$ distribution of the sum of squared residuals.
(c) Plot the likelihood as a function of θ over the range (-5,2).
(d) Explain why the expected value of θ given the data and posterior probability intervals for it cannot be calculated using a "flat prior" (i.e. treating the likelihood as proportional to the posterior pdf.) Is this problem special to the data in this problem, or will it be true for every possible pair of y and x vectors? (We assume that x observations will always be positive real numbers.)
(e) Calculate posterior 95% confidence intervals under the following priors for θ:
   (a) $N(0, 1)$
   (b) $N(1, 2)$
   (c) Cauchy, i.e. pdf
   $$\frac{1}{\pi(1+\theta^2)}\ .$$
   You will want to do this numerically, using Matlab or some similar program.