Stickiness[1]

by

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[1] I have benefited from discussion of this topic with a number of people, particularly Jordi Gali, Mark Gertler, and other participants in the macroeconomics seminar at NYU.

I. Introduction

Most prices and wages do not change daily, or even weekly, and many seldom change within a year. Keynes argued that this observation played a central role in an explanation of how an economy could operate for long periods of time at inefficiently low levels of activity and that it implied strong effects of monetary and fiscal policy on the level of economic activity. He gave little attention to the microeconomic foundations of wage stickiness, but proceeded to incorporate it directly into an equilibrium model of the economy as a whole, constructed as a collection of behavioral equations. The basic conclusion that wage and price stickiness is a source of non-neutrality in the effects of changes in nominal aggregate demand[2] is accepted even by many economists who do not agree with Keynes's diagnosis or prescriptions for aggregate fluctuations.

Modern practice is to build up macroeconomic models from analysis of behavior of individuals and to include stochastic elements of behavior in the theory explicitly. This allows study of interactions of behavior across different markets and avoids spurious or anomalous results arising from writing down behavioral equations directly, with stochastic terms tacked on as afterthoughts. But stickiness creates difficulties for this style of modeling. Competitive, market-clearing prices that are not sticky can be derived from equilibrium while being taken as exogenous in the modeling of every individual person or firm's behavior. But if prices are to be made sticky, we must either impose this behavior in an ad hoc way, not grounded in the modeling of individual behavior, or else model control of individual prices by individual firms or people. This might seem on the face of it to force abandonment of the macroeconomists' convention of working with single Y, P, and W variables, but by making a number of strong symmetry and functional form assumptions, it is possible to sidestep these complications. The resulting model, called the monopolistic competition with menu costs (MCMC) model and summarized recently in Goodfriend and King, has been argued to constitute a standard framework for macroeconomic research (see Blanchard comment). But there are other claimants to this title (see Kydland and Prescott) that are quite different.

Furthermore, the appeal of the MCMC setup appears to be primarily its tractability and its ability to deliver conventional conclusions. The microeconomic assumptions underlying it are clearly very far from anything that could be supported by appeal to the empirical microeconomic literature that studies price and wage setting, and there is slight attention in the literature to whether the aggregate implications of this model of stickiness match observed facts better than other specifications. Without further elaboration, the types of theoretical MCMC models found in the literature imply smooth time paths for price variables, because of the adjustment costs, and this tends also to lead to the conventional Keynesian idea that shocks to the economy impact quantities first, price and wage levels only with a lag. These kinds of implications have not been very carefully explored empirically.

In a recent paper (LSZ) I and my co-authors developed a weakly identified model of the US economy that included several price variables. The estimated impulse response dynamics from that model are reproduced in Figure 1. The figure confirms the stickiness of most price variables

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[2] I do not myself find "aggregate demand" to be a very useful, or even a very clearly definable, notion, but here I am trying to summarize conventional views.

in one sense: the *P* and *W* rows of the figure show smooth, slow responses to most types of disturbance, including monetary policy disturbances (which are the first two columns of the figure). However, in another sense they do not. The two largest source of disturbances to the wage and price time series are their "own shocks", on the diagonal of the figure, and these disturbances produce substantial instantaneous responses.[3] The own-shocks to *W* and *P* cannot be treated simply as "noise". The two variables are measured independently, yet their shocks have substantial cross effects on each other. This kind of behavior, combining smooth, slow reaction of prices and wages to most shocks, with strong, quick reaction to other shocks, does not emerge easily from MCMC style models, and the problems it raises have received little attention.

It is also worth noting that the important quantity variables in the model show the same pattern, with responses to policy and most other shocks smooth and slow, while a few shocks, particularly on the diagonal, produce strong, quick responses.

Another recent paper (Keating [1977]) has examined the joint dynamic behavior of a variety of price and wage time series, concluding that at both quarterly and monthly levels price and wage innovations are very weakly related.[4]

The macroeconomic implications of stickiness and the ability of models with stickiness to match the observed behavior of price and quantity data thus seem to be worth of further study. This study argues first that theory leaves the connection of micro-economic stickiness with aggregate non-neutrality of demand management policy a more open question than is perhaps commonly believed. It then goes on to discuss how well various approaches to modeling stickiness can be made to accord with the macroeconomic data on prices and quantities.

In section II there is a sticky wage model that implies wage dynamics mimicking that of a Taylor or Calvo specification, but with the exactly the same small aggregate demand effects (arising from real transactions costs) as in a purely classical model.[5] In section III, there is a "truly Keynesian", or "ISLM/Expectational-Phillips-Curve on its head"[6] model that includes

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[3] Since the model is not a reduced form, the correspondence between "left-hand-side" variables and equations is imprecise, but its form is close enough to that of a reduced form that (as can be seen) there is a strong tendency for the disturbances on the diagonal to account for substantial portions of the variation. (This is seen by comparing the size of the diagonal responses to that of other responses shown in the same row.)

[4] Keating concludes that price stickiness must play an important role in business cycle fluctuations. This paper will argue that his observations do not necessarily have the implications he draws from them.

[5] It is important to note that this is not an "implicit contracts" model. There is a widely accepted claim that, because wages are but one component of complicated, dynamic, implicit contracts between workers and employers, they should not be treated as guiding resource allocation. This leads to a prescription that wage data can be ignored in empirical work. This paper's neutral contracts, on the other hand, are explicit contracts, with their connection to agents' behavior laid out in the model.

[6] This label is justified by the model's taking expectations out of the Phillips curve, but treating them carefully in modeling investment and savings. The model thus has no "ISLM" component,

rational expectations and continuous clearing in asset markets, but disequilibrium in labor markets.[7] These first two models omit capital and investment. They are meant only to show how wide is the range of possible implications for macroeconomic policy of microeconomic observations of "price stickiness". Both models have sticky prices; yet one makes aggregate demand policy irrelevant to the evolution of the real economy, while the other makes it extremely powerful. In sections IV and V there are models including adjustment costs in capital, so that there is an investment function. The section IV model is purely classical, while that of section V adds "neutral stickiness" of the type introduced in section II. Here, though, prices as well as wages are sticky and the sticky prices enter the transactions demand for money. This model is meant to be rich enough to be (eventually) confronted with data.

We document a number of important characteristics of these models. In the models without stickiness, monetary contraction produces a discontinuous drop in prices. Even when monetary policy is specified in terms of an "interest rate rule" connecting the interest rate to inflation, monetary contraction need not cause any change in the interest rate. However, when the interest rate rule is given a form that implies an interest rate change when monetary policy contracts, it is easy to avoid any "liquidity puzzle" in these models. The liquidity puzzle is the problem that in many models without stickiness, monetary contraction implies a decline in nominal interest rates. This result emerges because contraction implies deflation, or a decline in inflation, and the nominal interest rate in such models is usually a real interest rate unresponsive to inflation, plus expected inflation. But with an interest rate rule, for plausible parameter values, monetary contraction implies an upward jump in interest rates as prices jump downwards. This is possible because the downward jump in prices is followed by a smooth return of the price level to an equilibrium path or level. Thus expected inflation jumps up at the same time that the price level jumps down. Of course this does still leave a puzzle: the data give no indication of jumping behavior in the price level.

If we suppose that price indexes like the CPI and the GDP deflator reflect primarily the behavior of contract prices like those in this paper's classical model with stickiness, we see the possibility of a more complete resolution of the puzzle. Because measured prices are a two-sided moving average of effective shadow prices, the effective prices could jump down, and at the same time be expected to start returning up to their long run path, even though measured prices show no downward jump, only a decline in the rate of change of prices.

In section IV it is shown that, as has already been noted in a few other papers in the literature, even in a model without any stickiness, the real effects of monetary policy are non-negligible, despite the absence of strong "Keynesian stickiness" effects and despite the fact that transactions costs are themselves very small. These effects do not show up in analysis of the model's steady state. They are effects of monetary shocks on the time path of investment. The effects are of an order of magnitude that suggests they could account for the entire real effect of monetary policy in estimated identified VAR models. In section V we find that, while the introduction of "neutral contracts" that enter the transactions measure has its main effect on price and wage behavior

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because expectations of future capital goods prices and real interest rates are critical to savings and investment behavior and are ignored in the ISLM abstraction.

[7] A very similar model appeared in Sims [1993].

(eliminating its "jumpiness"), it does affect the response of investment to monetary policy shocks, doubling it in the examples computed here.

There is some existing empirical work that fits equilibrium models with price and wage stickiness to macroeconomic data. Leeper and Sims [1994] achieved a fit quite a bit worse than produced by unconstrained VAR's, but substantially better than that of calibrated RBC models, for a model that included purely backward-looking wage and price stickiness. The best fit for that model implied little non-neutrality, despite the stickiness. Jinill Kim [1966] achieved a better fit, to a shorter list of variables (still including both real and nominal variables), with a model that implies stronger non-neutrality, using the MCMC framework.

The aim of this line of research is eventually to fit the models in sections IV and V and extensions of them that add other sorts of stickiness, to see how sharply the data speak, both on price-wage stickiness and on non-neutrality.

II. Sticky Wages that Leave Nominal Aggregate Demand Neutral

Explicit price and wage contracts of at least some forms could exist without implying economic behavior of real variables any different from what would be observed with continuously clearing spot markets. While the point seems obvious once one has understood it, there may not be another model in the literature that actually makes the point. The model below is kept as simple as possible, so that the stickiness mechanism and its effects are easily understood. It has stickiness in wages only, it has no capital, and it has no money. All three of these simplifications are relaxed in section V.

A. Consumers

1. Optimization Problem

$$\max_{C,L,\ell,B,M} \int_0^\infty e^{-\beta t} U(C, 1-L)\,dt \tag{1}$$

subject to

$$\lambda: \quad C\cdot(1+\gamma V) + \frac{\dot{B}+\dot{M}}{P} + \tau = \frac{rB}{P} + \frac{Y}{P} + \pi \tag{2}$$

$$\mu \quad \dot{Y} - w\dot{L} \le -\delta\cdot(Y - wL) \tag{3}$$

$$B \ge 0\,,\ \ M \ge 0 \tag{4}$$

$$V = \frac{PC}{M}\ . \tag{5}$$

Equations (2) and (5) describe an ordinary budget constraint with transactions costs, of the same type that has appeared Leeper and Sims [1994] and Sims [1994], for example. What is different about this model is (3), which arises from the contracting structure. To understand where it comes from, it is perhaps easiest to see that it can be derived from the following two constraints:

$$\dot{Y} \le w\ell - \delta Y \tag{6}$$

$$\dot{L} \geq \lambda - \delta L\ . \tag{7}$$

Here $\lambda(t)$ represents the rate at which new contracts are being signed at t. The rate of growth of total labor input is this rate, less the rate at which old contracts are expiring, given by δL. The rate of growth of the wage bill is the rate of addition of new contract payment obligations, $w\lambda$, less the rate of decay of old contract payment obligations, δY. The model makes sense if we use (6) and (7) explicitly in place of (3), but it makes more sense if we use (3) alone. This avoids any implication that Y and L must individually have differentiable time paths, i.e. that λ exists as a real-valued function of time. We may want to allow for the possibility that discrete lumps of contracts are signed at a given date, or in a stochastic version of the model that λ might behave like a white noise.

Note that (3) does not imply exponential return of $Y - wL$ to zero, because its left-hand side differs from $(d/dt)(Y - wL)$ by the term $-\dot{w}L$. Sudden changes in w can change $Y - wL$, but sudden changes in L cannot.

It is probably easiest to accept this as a stylized representation of wage contracting if we restrict ourselves to thinking of cases with $\lambda > 0$. Nonetheless, negative λ, which we must allow if we are contemplating possible white-noise-like behavior for λ, can be interpreted. It implies that contracts are being deliberately ended at a rate greater than the exogenous decay rate δ. For these terminations to be valued at a market flow rate w unrelated to the history of contracting implies a kind of "voluntary separation" interpretation. Workers who leave retain severance pay, retirement benefits, or the like that compensate them for the difference in value between their existing contract with the firm and what they can obtain in a new contract at the market rate. There are examples of layoffs that have this character ("employee buyouts", "early retirement incentives"), though of course not all of them do.

2. FOC s

∂C:
$$D_1U(C,1-L) = \lambda \cdot (1 + 2\gamma V) \tag{8}$$

∂L:
$$D_2U(C,1-L) = -w\dot{\mu} - \dot{w}\mu + (\beta + \delta)\mu \tag{9}$$

∂Y:
$$\frac{\lambda}{P} = -\dot{\mu} + (\beta + \delta)\mu \tag{10}$$

∂B:
$$-\frac{\dot{\lambda}}{P} = \left(r - \beta - \frac{\dot{P}}{P}\right)\frac{\lambda}{P} \tag{11}$$

∂M:
$$-\frac{\dot{\lambda}}{P} + \left(\beta + \frac{\dot{P}}{P}\right)\frac{\lambda}{P} = \gamma\frac{\lambda}{P}V^2 \tag{12}$$

B. Firms

1. Optimization Problem

$$\max_{\pi,\lambda,L,Y,B} \int_0^\infty e^{-\beta t}\phi(\pi_t)\,dt \tag{13}$$

subject to

$$\xi: \qquad \pi = f(L) - \frac{Y}{P} + \frac{\dot{B}^F}{P} - \frac{rB^F}{P} \tag{14}$$

$$\nu: \qquad \dot{Y} - w\dot{L} \geq -\delta(Y - wL) \tag{15}$$

$$B \geq 0\ . \tag{16}$$

Notice that the firms face the same contracting constraint in (15) that consumers do in (3).

2. FOC s

$$\partial\pi: \qquad \phi' = \xi \tag{17}$$

$$\partial L: \qquad \xi f' = -\dot{\nu}w - \nu\dot{w} + (\beta+\delta)\nu w \tag{18}$$

$$\partial Y: \qquad \frac{\xi}{P} = -\dot{\nu} + (\beta+\delta)\nu \tag{19}$$

$$\partial B: \qquad -\frac{\dot{\xi}}{P} + \left(\beta + \frac{\dot{P}}{P}\right)\frac{\xi}{P} + = r\frac{\xi}{P} \tag{20}$$

C. Government

1. Constraint

$$\dot{B} - \dot{B}_F + \dot{M} + \tau P = r(B - B_F) \tag{21}$$

2. Behavior

The government must set two dimensions of the behavior of B, τ, M, r, and P. Following what has been the convention in macroeconomics, we will at first assume that it fixes exogenously a time path for M and chooses a policy for the primary surplus τ that prevents real debt from exploding.[8]

[8] As pointed out in Sims [1997], in a model like this there are, even with this fixed-M-path policy, multiple equilibria. However because we have specified the technology so that consumption is zero when real balances are zero, all the equilibria except the stable one on which we focus attention make real balances vanish in finite time, pushing consumption to zero in the process. No automatic market arbitrage mechanism will keep the economy off such a path. Since these equilibria are such bad outcomes, though, it seems reasonable to suppose that they are precluded

D. *Model without stickiness*

The model without stickiness, to which we will compare this model with sticky wages, is nearly the same. We replace the Y terms in the consumer and firm budget constraints with WL, where W is the spot wage rate, and drop the equations defining the contract structure, (3) and (15) to arrive at, as FOC's for individuals

$$\partial C: \qquad D_1 U = (1 + 2\gamma V)\frac{\lambda}{P} \qquad (22)$$

$$\partial L: \qquad D_2 U = \lambda \frac{W}{P} \qquad (23)$$

and for firms

$$\partial L: \qquad \zeta f' = \zeta \frac{W}{P}\ . \qquad (24)$$

The other FOC's, other than those dropped, are unchanged.

The FOC's with respect to Y then disappear, and the right hand side of the FOC's with respect to L become simply $\lambda W/P$, in the case of (9), and $\zeta W/P$ in the case of (18). In other words

$$D_2 U(C, 1-L) = \lambda \frac{W}{P} \qquad (25)$$

$$\zeta f' = \zeta \frac{W}{P}\ . \qquad (26)$$

E. *Analysis*

The models with and without stickiness have the same solutions for any given time path of M. There are several ways to see why this is true. One is to note that equations (8)-(12) and (18)-(20) in the model with sticky wages play the same role, in terms of their implications once Lagrange multipliers are solved out, as the smaller set of corresponding equations (22)-(24), (12), and (20) in the model without stickiness, namely to produce the relationships

$$\frac{D_2 U \cdot (1 + \gamma V)}{D_1 U} = f'(L) \qquad (27)$$

and

$$-\frac{\frac{d}{dt} D_1 U}{D_1 U} + \frac{\gamma \dot{V}}{1 + \gamma V} = r - \beta - \frac{\dot{P}}{P}\ . \qquad (28)$$

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by some backstop mechanism, e.g. a commitment by fiscal authorities to tax to back the value of the currency if the value reaches some extreme low level.

Of course while it is obvious that (27) and (28) are implied by the FOC's of the model without stickiness, that they follow from the model with stickiness requires some argument, which we will lay out below. For now, though, note that these two equations, together with (5) (the definition of V), the liquidity preference relation

$$r = \gamma V^2 \tag{29}$$

(implied by (11) and (12)) and the social resource constraint

$$C \cdot (1 + \gamma V) = f(L)\,, \tag{30}$$

(which is implied by the private constraints (2) and (14) and the government constraint (21)) determine the five variables C, L, V, r, and P from a given policy-determined path for M. Again, further argument is required to show that there is a unique stable solution to the system, and we will lay the argument out below. But accepting for now that there is a unique stable solution, we see that this system has the same form for the models with and without stickiness. In other words, the mapping from the time path of M to the time path of the five variables C, L, V, r, and P is the same in both models.

Note that the system (27)-(30) does not involve the level of P, only $\dot{P}/P$, and that M enters the system only as a ratio to P, in (5). Thus given any solution for a given path $\{M_t\}_{t=0}^{\infty}$, we know that a solution for the rescaled path $\{\kappa M_t\}_{t=0}^{\infty}$ is obtained by changing the original P path to κP while keeping r, C, V, and L unchanged. Money is therefore neutral. It is not super-neutral, because changing the rate of growth of P generally changes r, therefore (by (29)) V and therefore C and L. The important point is that both these statements are true, in exactly the same way, whether prices are "sticky" or not.

We still need to verify that (8)-(12) and (18)-(20) do reduce to (27)-(30). Since (11) and (20) have the same form, they imply that

$$\frac{\xi_t}{\lambda_t} = \frac{\xi_0}{\lambda_0}\,, \tag{31}$$

all t. From this point on we will assume that the solution involves stable time paths for μ and ν. It is possible under mild regularity conditions to prove that these Lagrange multipliers have stable paths, but the argument becomes quite technical, and is thus omitted.[9] Equations (10) and (19) both have unique stable solutions, of the form

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[9] We are in effect verifying that if the model has a stable solution, then it has a stable solution that is the same for the sticky wage and non-sticky wage models. The argument we make explicitly here does not rule out the possibility that there might be solutions to the sticky wage model that involve unstable paths for the Lagrange multipliers and correspond to no equilibrium of the non-sticky price model. However, as noted in the text, it is possible to rule this out under mild regularity conditions.

$$\mu(t) = \int_0^\infty e^{-(\beta+\delta)s} \frac{\lambda_{t+s}}{P_{t+s}} ds \tag{32}$$

$$\nu(t) = \int_0^\infty e^{-(\beta+\delta)s} \frac{\zeta_{t+s}}{P_{t+s}} ds \ , \tag{33}$$

and because of (31) we know that this implies that

$$\nu \equiv \frac{\zeta_0}{\lambda_0}\mu \ . \tag{34}$$

But then from (8)-(10) we get

$$\frac{D_2U\cdot(1+2\gamma V)}{D_1U} = \frac{-\dot{\mu}+(\beta+\delta)\mu}{\lambda} = -\frac{\dot{w}}{w}\frac{\mu}{\lambda} + \frac{-\dot{\mu}w+(\beta+\delta)\mu w}{\lambda} = -\frac{\dot{w}}{w}\frac{\mu}{\lambda} + \frac{w}{P} \tag{35}$$

while from (18)-(19) we get

$$f' = -\frac{\dot{w}}{w}\frac{\nu}{\zeta} + \frac{-\dot{\nu}w+(\beta+\delta)\nu w}{\zeta} = -\frac{\dot{w}}{w}\frac{\nu}{\zeta} + \frac{w}{P} \ . \tag{36}$$

From (31) and (34) we know that the right-hand sides of (35) and (36) are the same, so we arrive at (27), which was our target. It is easy to see that (28) is derived in exactly the same way in both models.

F. Discussion

In this economy the wage on new contracts, *w*, is forward-looking. We can see from (36) that it can be represented as a weighted average of expected future marginal products (in dollars) of labor. At the same time, the aggregate economy-wide average wage, Y/L, is by (6) and (7) (heuristically, since these equations are not actually part of the model) a weighted average of current and past *w*'s, which makes it a two-sided moving average of past and expected future marginal products (in dollars) of labor. This is exactly the form of wage dynamics postulated in the overlapping contract model of Taylor and the randomly-timed-adjustment model of Calvo. How is it that this model can mimic the price behavior of these other models, without producing their conclusion that stickiness generates non-neutrality?

In this economy, a labor contract promises a fixed stream of labor hours (subject to a randomly timed termination) in return for a fixed stream of dollar payments. The model contains a capital market that can produce present values for streams of future payments. The representative family, when deciding whether to increase or decrease *L*, considers by how much the present value of the stream of payments it obtains by increasing *L* by one unit exceeds the present value of the stream of future labor hours it is promising. This difference is the dollar return to decreasing leisure by one unit at this instant. It plays the same role as does the nominal wage in a model with spot labor markets.

Now suppose there is an unanticipated shift in *M*, resulting in a new equilibrium with a higher price level. Surely, it would seem, the outstanding stock of wage contracts, negotiated under different expectations, will produce real effects in the wake of this policy action. In a sense there are real effects. The representative family takes a capital loss on its old labor contracts, as these promise labor at a lower real rate of compensation than was originally anticipated. But this capital loss is the other side of a capital gain for the representative firm, which can now pay labor a lower than anticipated real rate of compensation. In this economy, the representative firm is owned by the representative family, which receives dividends from it. Thus these offsetting capital gains and losses have no effect on the representative family's budget constraint. And because the marginal compensation to changes in L is entirely a forward-looking object, the offsetting capital gains and losses produce no effect on labor supply or demand.

This result is a little like Ricardian Equivalence in that, on the one hand, it requires strong assumptions that are easy to criticize, while, on the other hand, it provides a limiting case that changes the way we think about macroeconomic issues. Few people think that Ricardian Equivalence holds even as a first-order approximation. Yet the principle that underlies it suggests that the effects of fiscal policy could well be much weaker than naïve approaches that do not trace out complete general equilibrium effects would suggest. In this model of "neutral stickiness", perfect insurance and perfect capital markets are important maintained assumptions, as for Ricardian Equivalence. It is unlikely that the model is a good approximation to the truth. But by showing that the degree of stickiness of prices has no necessary connection to the amount of non-neutrality in the economy, it may change our thinking. With this model in mind, we see that no amount of micro-economic evidence for slowly or infrequently changed prices and wages can, by itself, tell us how important non-neutrality might be in the economy.

G. Robustness to form of contract and extension from wage to price stickiness

The essential feature of the contracts in this model, that leads to their not affecting neutrality, is that though they fix a "price" they do not represent open-ended commitments to sell at that price. Individual workers are working or not. If their contract specifies a wage that turns out to have been high, they benefit from this, and if their wage turns out to have been low, they lose. But these are changes in their wealth, not in effective slopes of their budget lines.

That there are no effects at all in this model depends on the assumption that there is a representative family that owns a representative firm, so that wealth redistribution between family and firm does not affect even the level of the budget line. More generally, with incomplete markets and nominal contracts of this type there would be wealth redistribution as the result of unanticipated price level changes, and this would have some real effect. The aggregate effects through this channel alone would probably be modest and of ambiguous sign, however.

Though wages are more often thought of as set by contracts something like those we describe here, many prices are also set along these lines. Catalogs, which offer goods at a price that is often fixed explicitly for a certain span of time, usually represent contracts of this type, because they usually do not represent a commitment to supply the goods even if current stock runs out, and the goods being offered usually have some durability, so that both seller and buyer have some capacity to hold inventories. Thus if the market price moves much above that in the catalog, the stock of the good sells out, the good is unavailable from the catalog, and the effective shadow price is unaffected by the "fixed" catalog price. The rise in the market price has generated a wealth loss for the catalog issuer and a wealth gain for the consumers who snapped up the stock at the low catalog price. Modifying the model so that expenditures on goods, as well as income from wages, occur in fixed-price contracts that preserve neutrality, is straightforward.

III. A Truly Keynesian Model

A. *Consumers*

1. Optimization Problem

$$\max_{C,B,M} \int_0^\infty U(C_t, 1-L_t) e^{-\beta t} dt \qquad (37)$$

subject to

$$PC \cdot (1+\gamma V) + \dot{B} + \dot{M} + \tau = WL + \pi \qquad (38)$$

$$B \geq 0, \quad M \geq 0 \qquad (39)$$

2. FOC s

∂C: $$D_1 U = \lambda P \cdot (1+2\gamma V) \qquad (40)$$

∂B: $$-\frac{\dot{\lambda}}{\lambda} = r - \beta \qquad (41)$$

∂M: $$-\frac{\dot{\lambda}}{\lambda} = \gamma V^2 - \beta \qquad (42)$$

∂L: (not used directly) $$D_2 U = \lambda W . \qquad (43)$$

B. *Firms*

1. Optimization Problem

$$\max_{\pi, B_F} \int_0^\infty \phi(\pi_t) e^{-\beta t} dt \qquad (44)$$

subject to

ζ: $$\pi = Pf(L) - WL + \dot{B}_F - rB_F \ . \qquad (45)$$

2. FOC s

$\partial \pi$: $$\phi' = \lambda \qquad (46)$$

∂L: (not used directly) $$f' = \frac{W}{P} \ . \qquad (47)$$

∂B_F: $$-\frac{\dot{\zeta}}{\zeta} = r - \beta \qquad (48)$$

C. Government

1. Constraint

$$\dot{B} - \dot{B}_F + \dot{M} + \tau P = r \cdot \left(B - B_F\right). \tag{49}$$

2. Behavior

As in the other models we look at, the government has to choose two of the five quantities $B - B_F$, r, P, τ and M, leaving the others to be determined by the government budget constraint and private behavior. We will consider a variety of possible specifications for government behavior.

D. Price and Wage Adjustment

Phillips curve:

$$\frac{\dot{W}}{W} = \theta \cdot \left(\frac{D_1 U}{D_2 U} - \frac{W}{P} \right) \tag{50}$$

Markup equation:

$$\frac{\dot{P}}{P} = \zeta \cdot \left(\frac{W}{P} - f' \right) \tag{51}$$

IV. Classical Model with an Investment Function

A. Consumer

1. Optimization problem

$$\max \int_0^\infty e^{-\beta t} \, \frac{C_t^{\mu_0} (1-L)^{\mu_1}}{\mu_0 + \mu_1} \, dt \tag{52}$$

subject to

$$PC \cdot \left(1 + \gamma V\right) + P_K I + \dot{B}_C + \dot{M} + \tau = WL + hK + \pi + rB_C \tag{53}$$

$$\dot{K} = I - \delta K \tag{54}$$

$$V = \frac{PC}{M} \, . \tag{55}$$

Choose C, L, V, M, I, B_C, and K paths.

2. FOC s

$$\frac{C}{1-L} = \frac{\mu_0 w}{\mu_1 \cdot \left(1 + 2\gamma V\right)} \tag{56}$$

$$r - \beta - \frac{\dot{P}}{P} = \left(1 - \mu_0\right) \frac{\dot{C}}{C} + \mu_1 \frac{\dot{L}}{1-L} + \frac{2\gamma \dot{V}}{1 + 2\gamma V} \tag{57}$$

$$r = \gamma V^2 \tag{58}$$

$$\frac{h}{P_K} = r + \delta - \frac{\dot{P}_K}{P_K} \ . \tag{59}$$

B. Firm

1. Optimization problem

$$\max \int_0^\infty e^{-\beta t} \phi\left(\pi_t\right) dt \tag{60}$$

subject to

ζ: $$\pi = P_K I + PC^* - WL - hK + \dot{B}_F - rB_F \ . \tag{61}$$

ω: $$C^* + \left(1 + \xi \frac{I}{K}\right) I = AK^\alpha L^{1-\alpha} \tag{62}$$

Choose π, I, K, L, C^* and B_F paths. Our formulation here is slightly unconventional, in that the investment dynamics occur in the consumer's problem, while the costs of adjustment appear in the firm's problem. This makes P_K / P play the role of "*q*".

2. FOC s

∂I: $$P_K \zeta = \left(1 + 2\xi \frac{I}{K}\right) \omega \tag{63}$$

∂K: $$h\zeta = \omega \left(\alpha \cdot A \cdot \left(\frac{K}{L}\right)^{\alpha - 1} + \xi \frac{I^2}{K^2} \right) \tag{64}$$

∂L: $$\zeta W = \omega \cdot (1 - \alpha) \cdot A \cdot \left(\frac{K}{L}\right)^\alpha \tag{65}$$

∂B_F: $$-\frac{\dot{\zeta}}{\zeta} = r - \beta \tag{66}$$

∂C^*: $$P\zeta = \omega \ . \tag{67}$$

Market clearing then requires that

$$C^* = \left(1 + \gamma V\right) C , \tag{68}$$

so that produced consumer goods match consumption, including transaction costs.

C. Government

1. Constraint

$$\frac{\dot{B}_C - \dot{B}_F + \dot{M}}{P} + \tau = r\frac{B_C - B_F}{P} \ . \tag{69}$$

2. Behavior

(Government behavior is the same in this model and that of the following section.)

V. Classical Model with Stickiness

Here we combine contract-stickiness as in the model of section II, but now in both wages and prices, with the classical model of section IV. In section II we did not include money. When we have real balances, and transactions costs, in a model where transactions prices are not true shadow prices, there is some ambiguity as to how to define real balances. It seems most natural to use a flow of actual contract payments to measure transactions, not a notional value of production or purchases priced at shadow prices. This does complicate the model, however, and eliminates the neat conclusion that contract prices have no effect on the real equilibrium.

A. Consumer

1. Optimization problem

$$\max \int_0^\infty e^{-\beta t} \frac{C_t^{\mu_0}(1-L)^{\mu_1}}{\mu_0 + \mu_1} dt \tag{70}$$

subject to

λ: $$Y_C + P_K I + \dot{B}_C + \dot{M} + \tau = Y_L + hK + \pi + rB_C \tag{71}$$

ν_K: $$\dot{K} = I - \delta K \tag{72}$$

ψ_V: $$V = \frac{Y_C}{M} \ . \tag{73}$$

ψ_C: $$C^* \geq C \cdot (1 + \gamma V)$$

ν_{YC}: $$\dot{Y}_C - p\dot{C}^* \geq -\eta_C \cdot \left(Y_C - pC^*\right) \tag{74}$$

ν_{YL}: $$\dot{Y}_L - w\dot{L} \leq -\eta_L \cdot (Y_L - wL) \tag{75}$$

$$B_C \geq 0,\ M \geq 0 \ .$$

Two definitional equations used to generate transactions price and wage indexes, but not part of firm or consumer optimization problems, are

$$P \cdot C^* = Y_C \tag{76}$$

$$W \cdot L = Y_L \ . \tag{77}$$

The consumer chooses C, L, V, M, I, B_C, P, W, Y_C, Y_L, and K paths.

2. FOC s

∂C: $$\frac{\mu_0}{\mu_0+\mu_1} C^{\mu_0-1}(1-L)^{\mu_1} = \psi_C \cdot (1+2\gamma V) \quad (78)$$

∂C^*: $$-\dot{p}\nu_{YC} - p\dot{\nu}_{YC} + (\beta+\eta_{YC})p\nu_{YC} = \psi_C$$

∂Y_C: $$\lambda = -\dot{\nu}_{YC} + (\beta+\eta_{YC}) - \frac{\psi_V}{M} \quad (79)$$

∂L: $$\frac{\mu_1}{\mu_0+\mu_1} C^{\mu_0}(1-L)^{\mu_1-1} = -w\dot{\nu}_{YL} - \dot{w}\nu_{YL} + (\beta+\eta_L)w\nu_{YL} \quad (80)$$

∂Y_L: $$-\dot{\nu}_{YL} + (\beta+\eta_L)\nu_{YL} = \lambda \quad (81)$$

∂K: $$-\dot{\nu}_K + (\beta+\delta)\nu_K = h\lambda \quad (82)$$

∂I: $$P_K\lambda = \nu_K \quad (83)$$

∂B_C: $$-\frac{\dot{\lambda}}{\lambda} = r-\beta \quad (84)$$

∂M: $$r\lambda = \frac{\psi_V Y_C}{M^2} \quad (85)$$

B. Firm

1. Optimization problem

$$\max\int_0^\infty e^{-\beta t}\phi(\pi_t)dt \quad (86)$$

subject to

ζ: $$\pi = P_K I + Y_C - Y_L - hK + \dot{B}_F - rB_F\ . \quad (87)$$

ω: $$C^* + \left(1+\xi\frac{I}{K}\right)I \le AK^{\alpha}L^{1-\alpha} \quad (88)$$

σ_{YL}: $$\dot{Y}_L - w\dot{L} \le -\eta_L\cdot(Y_L - wL) \quad (89)$$

σ_{YC}: $$\dot{Y}_C - p\dot{C}^* \ge -\eta_C\cdot(Y_C - pC^*) \quad (90)$$

$$B_F \ge 0$$

2. Firm FOC s

$\partial\pi$: $$\zeta = \varphi'(\pi) \quad (91)$$

∂C^*: $$\omega = -p\dot{\sigma}_{YC} - \dot{p}\sigma_{YC} + (\beta + \eta_C)\sigma_{YC}p \tag{92}$$

∂Y_L: $$\zeta = -\dot{\sigma}_{YL} + (\beta + \eta_L)\sigma_{YL} \tag{93}$$

∂L: $$-w\dot{\sigma}_{YL} - \dot{w}\sigma_{YL} + (\beta + \eta_L)w\sigma_{YL} = \omega(1-\alpha)K^{\alpha}L^{-\alpha} \tag{94}$$

∂I: $$P_K\zeta = \omega \cdot \left(1 + 2\xi\frac{I}{K}\right) \tag{95}$$

∂K: $$h\zeta = \omega \cdot \left(\alpha K^{\alpha-1}L^{1-\alpha} + \xi\frac{I^2}{K^2}\right) \tag{96}$$

∂B_F: $$\frac{-\dot{\zeta}}{\zeta} = r - \beta \tag{97}$$

C. Government

The government constraint is exactly as in section II. We experiment with various specifications of government behavior, as specified below in the discussion of results.

VI. Model Comparison Results

Figure 2 below shows impulse responses in the section IV model to a monetary policy shock. Note that there is no liquidity puzzle: Interest rates fall, *M* expands, investment and consumption rise. There is a price puzzle. This apparent monetary expansion produces a slight fall in the price level. In Figure 3 we show the result of conducting the same experiment in a model differing only in that it has neutral contracts. The price effects of the expansion are now almost zero, while the effects on investment have been greatly increased. The investment increase is the same order of magnitude as the initial rise in *M* (all variables are in log units except *r*, which is in natural units at an annual rate). In the VAR model of Figure 1, the peak investment response is somewhat larger in logs than the peak *M1* response, but the effects are in the same direction and of the same order of magnitude. In this model, steady-state transactions costs are 1.4 per cent of steady-state consumption, so this result occurs despite transactions costs being a very small part of total economic activity.

These impulse responses are still not ready for statistical testing as displayed, of course. The discontinuous initial response of *M* does not match the data, where *r* moves instantly but *M* does not. Also, the relative size of the *r* response is small relative to the *M* response. It is possible to get rid of most of these undesirable characteristics of the responses one at a time, by adjusting the policy rule and the persistence of the shocks to it. However, in a model this complex, getting rid of all the undesirable characteristics of the responses at once by trial and error is not practical. The next step should be letting the computer work on improving the fit.

[The presentation will include display and discussion of responses for the simpler models in earlier sections, including the "truly Keynesian" model, and will show and discuss some further response graphs for the section IV and V models.]

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LSZ

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Figure 1

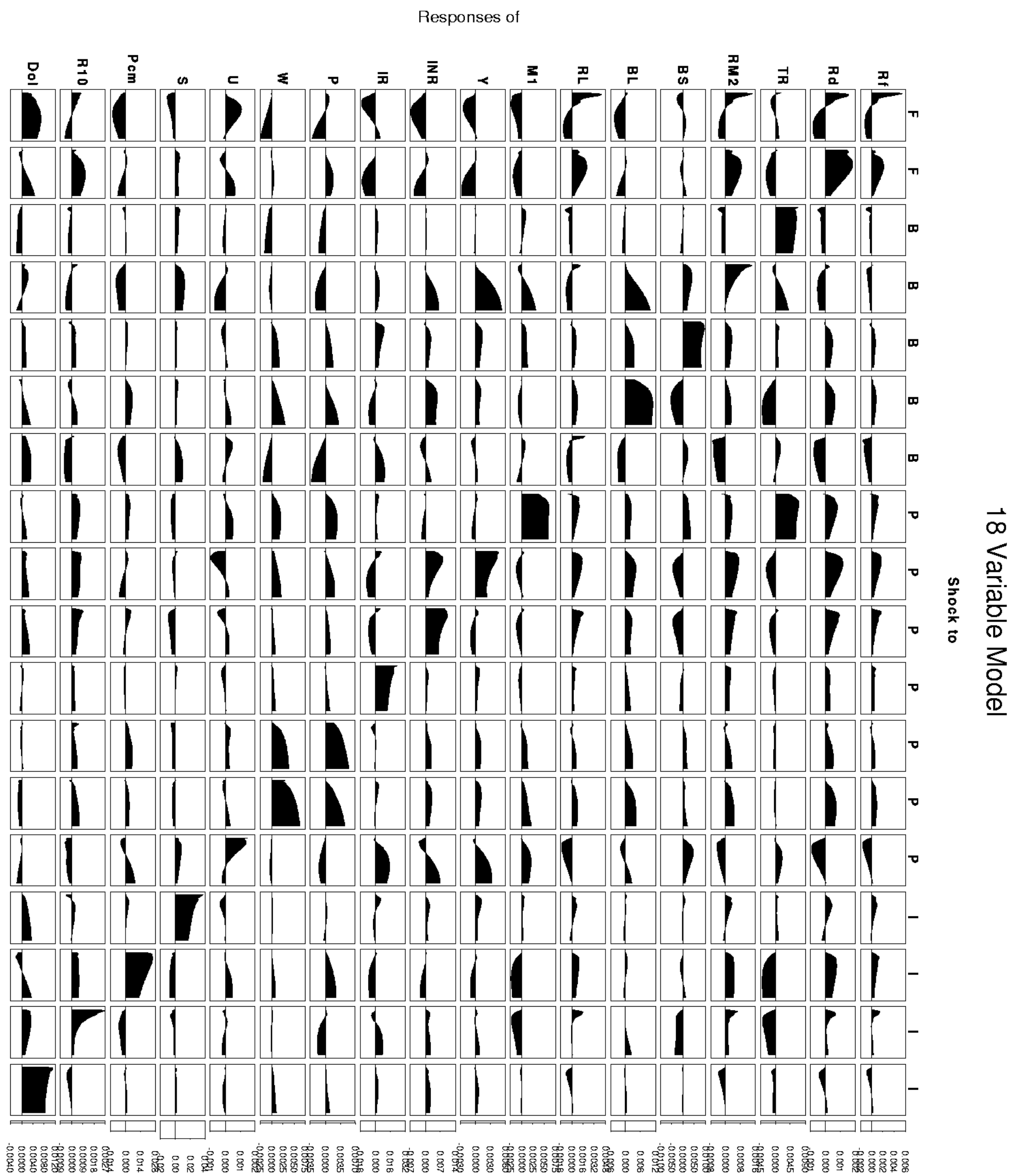

18 Variable Model
Shock to
Responses of
Dol
R10
Pcm
S
U
W
P
IR
INR
Y
M1
RL
BL
BS
RM2
TR
Rd
Rf

Figure 2

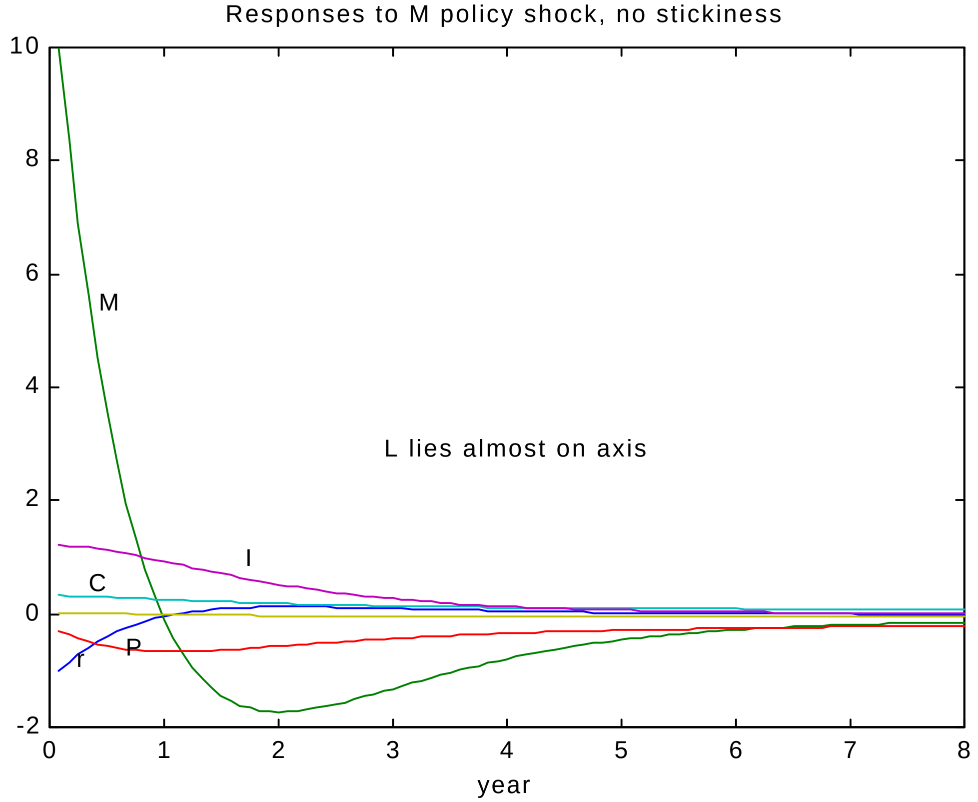
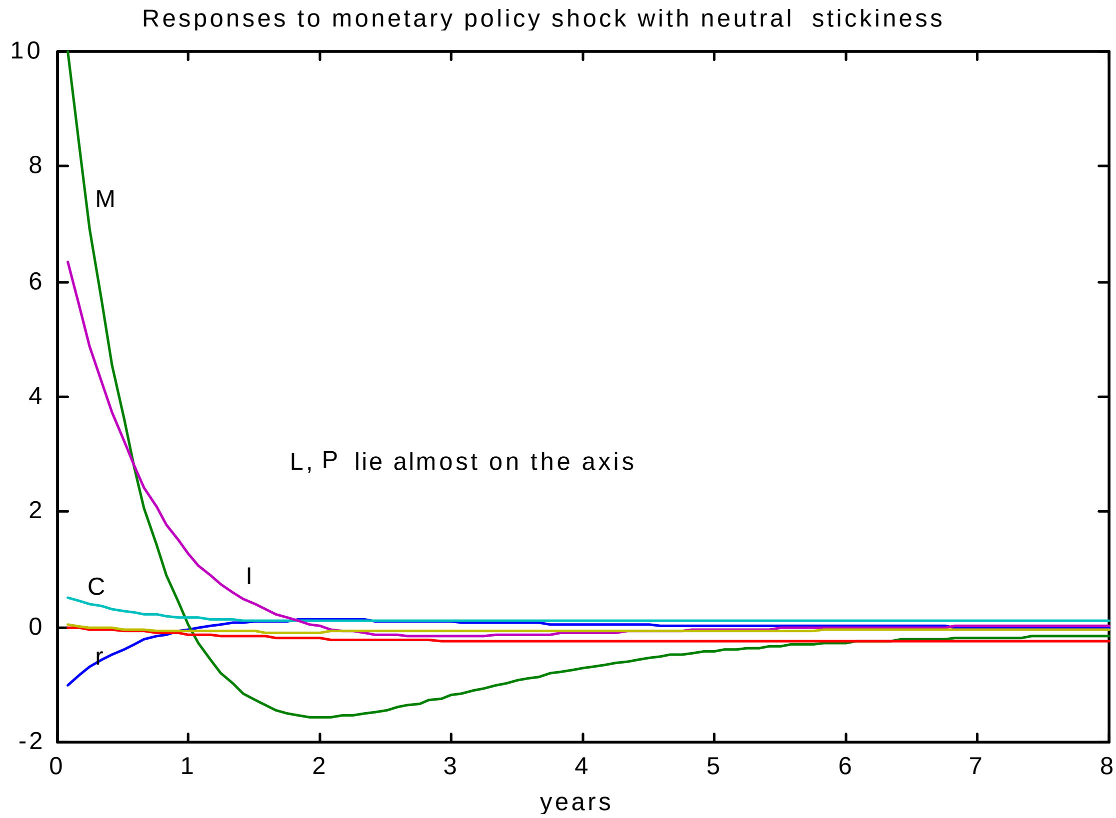
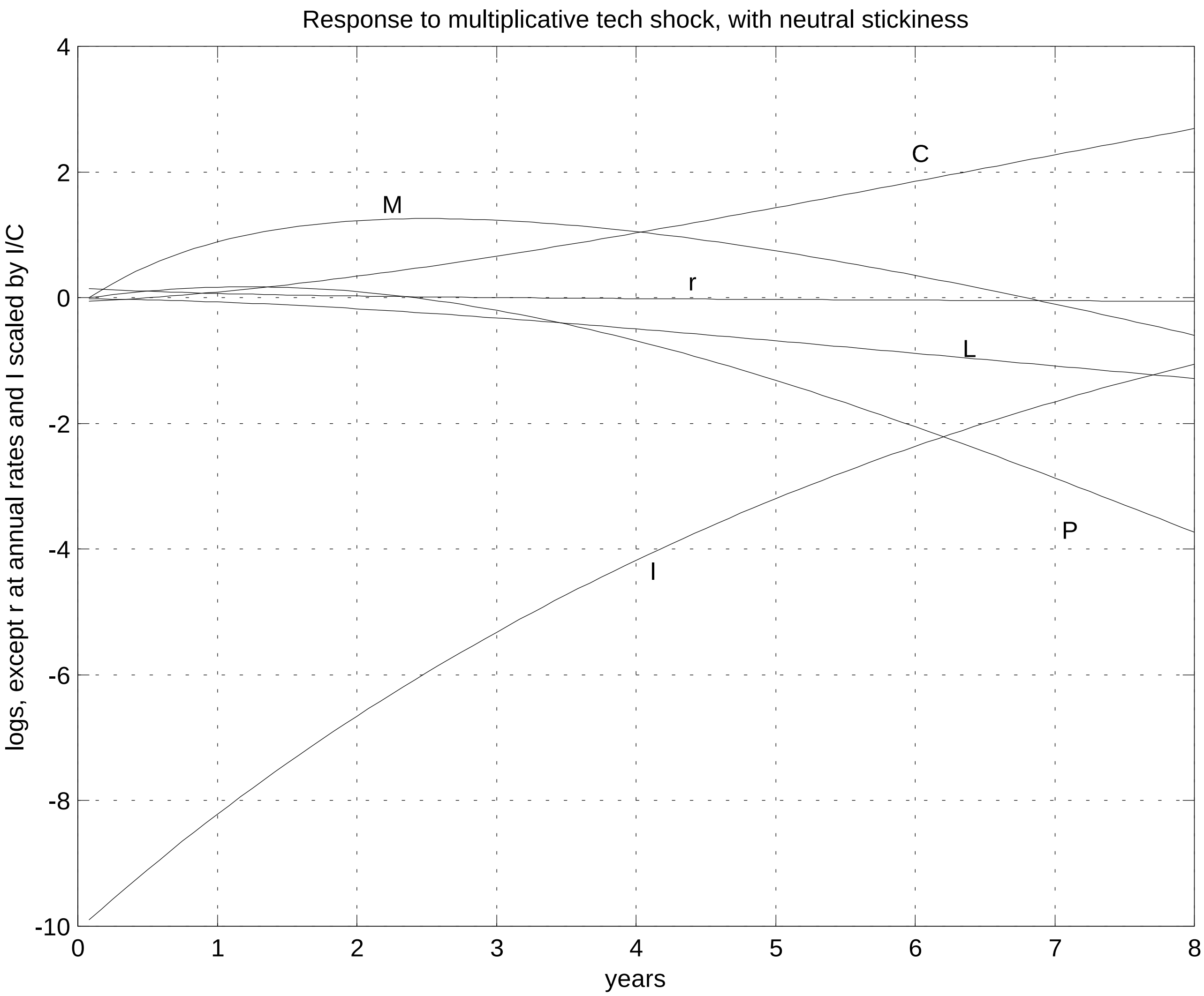
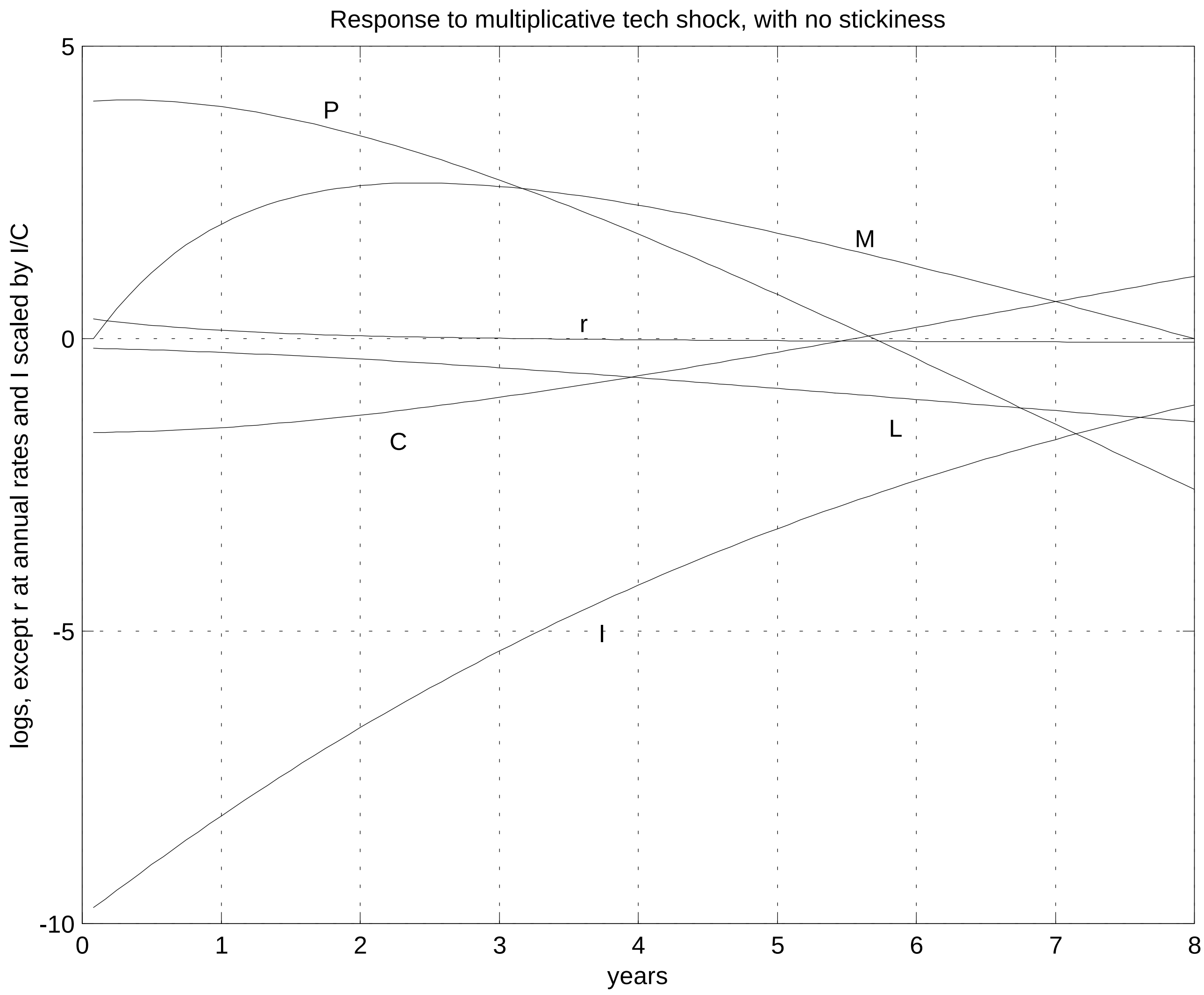

Responses to M policy shock, no stickiness
10
8
6
4
2
0
-2
M
L lies almost on axis
I
C
P
r
0
1
2
3
4
5
6
7
8
year

Figure 3

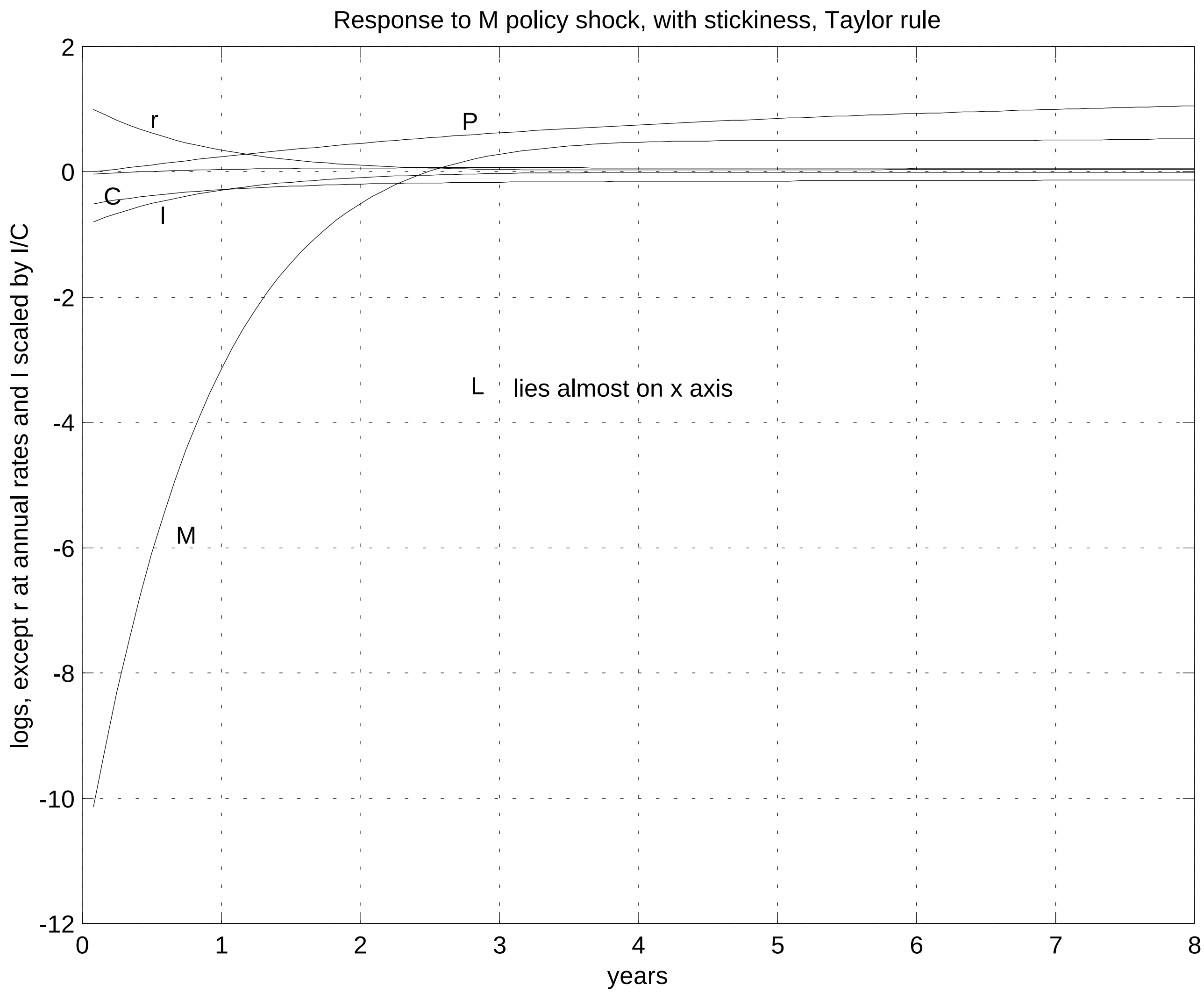


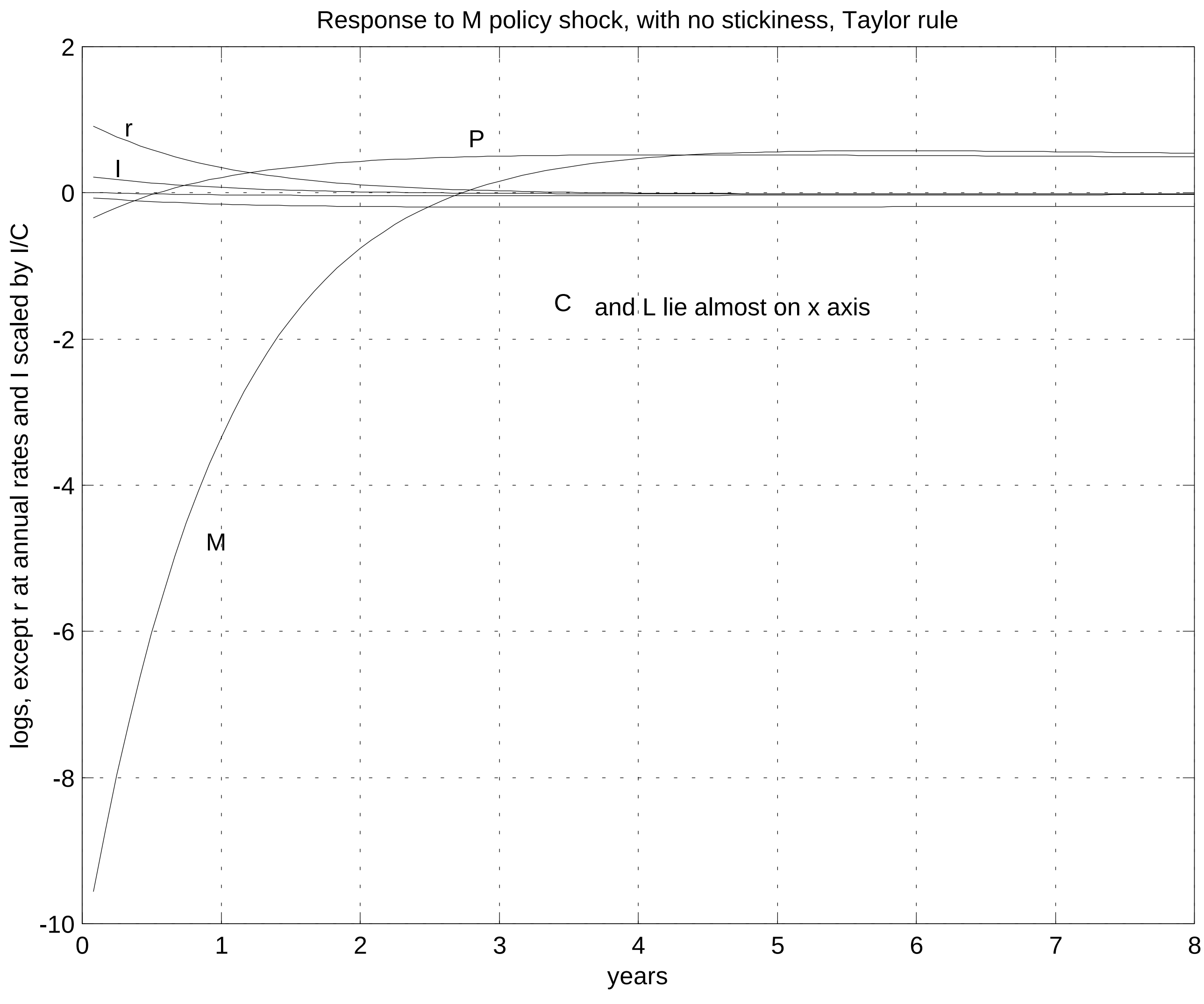
Responses to monetary policy shock with neutral stickiness
10
8
6
4
2
0
-2
M
L, P lie almost on the axis
C
I
r
0
1
2
3
4
5
6
7
8
years


Response to multiplicative tech shock, with neutral stickiness
4
2
0
-2
-4
-6
-8
-10
0
1
2
3
4
5
6
7
8
years
logs, except r at annual rates and I scaled by I/C
C
M
r
L
P
I

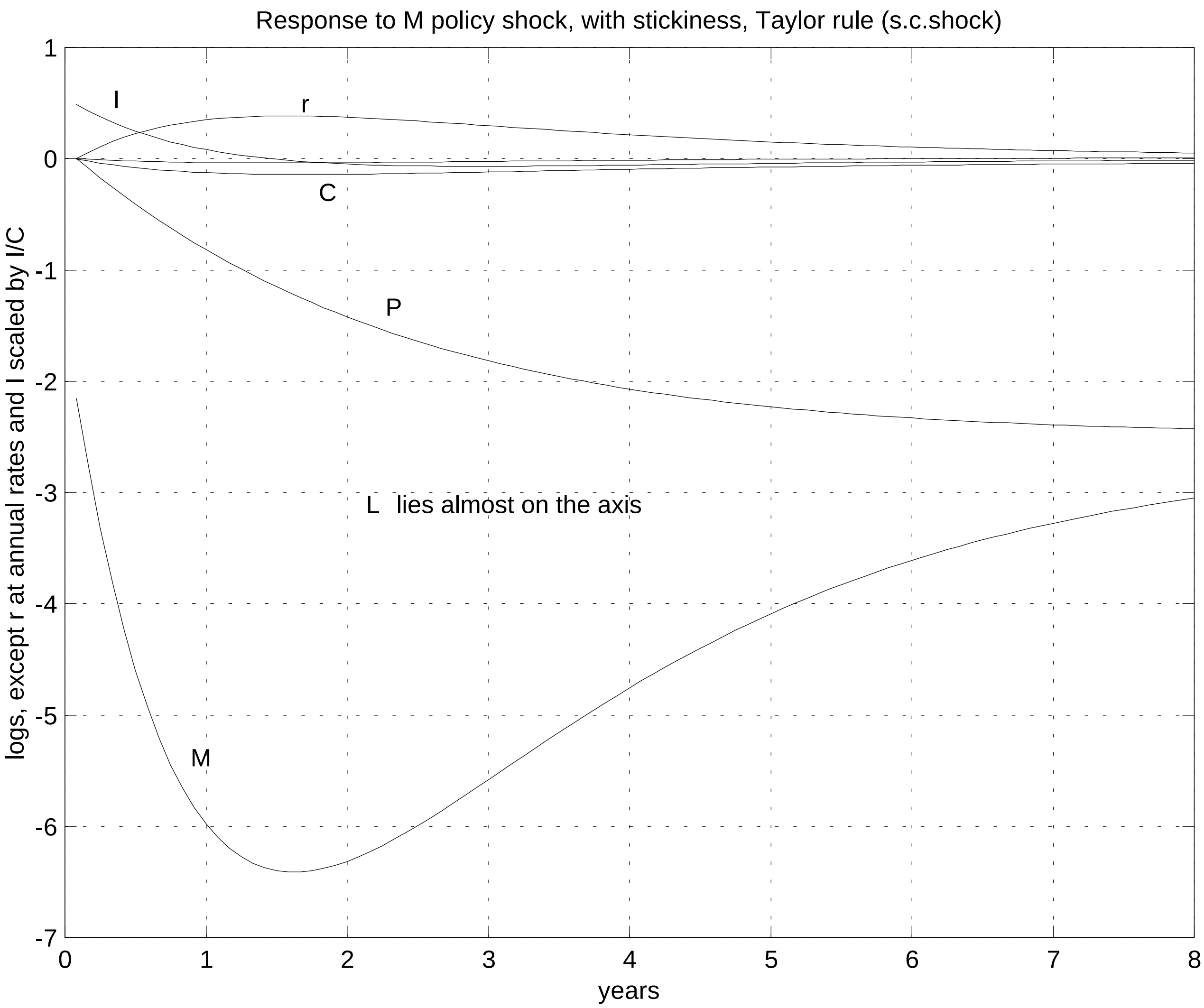


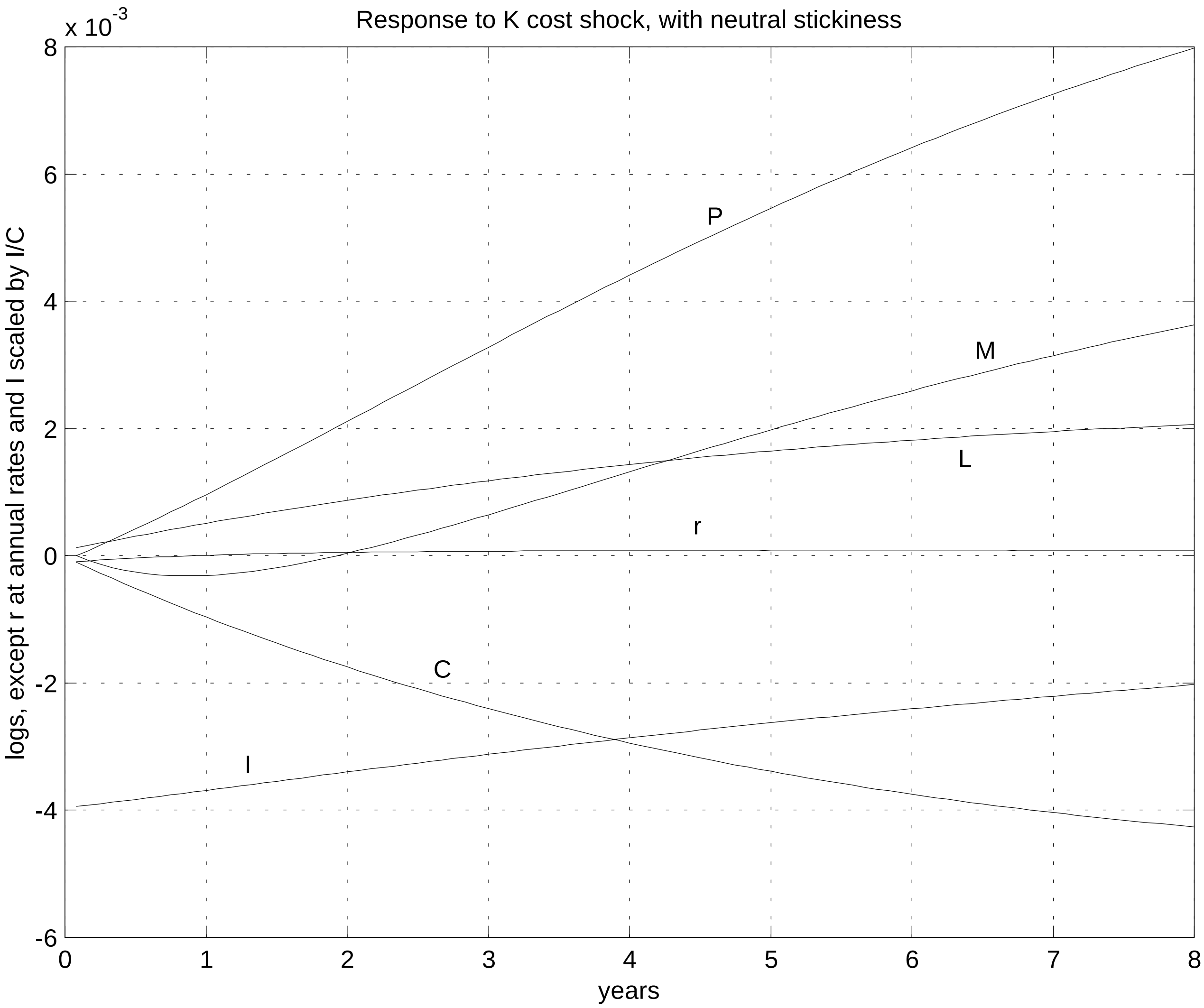
Response to multiplicative tech shock, with no stickiness
logs, except r at annual rates and I scaled by I/C
years
P
M
r
C
L
I
5
0
-5
-10
0
1
2
3
4
5
6
7
8


Response to M policy shock, with stickiness, Taylor rule
2
0
-2
-4
-6
-8
-10
-12
0
1
2
3
4
5
6
7
8
years
logs, except r at annual rates and I scaled by I/C
r
P
C
I
L
lies almost on x axis
M

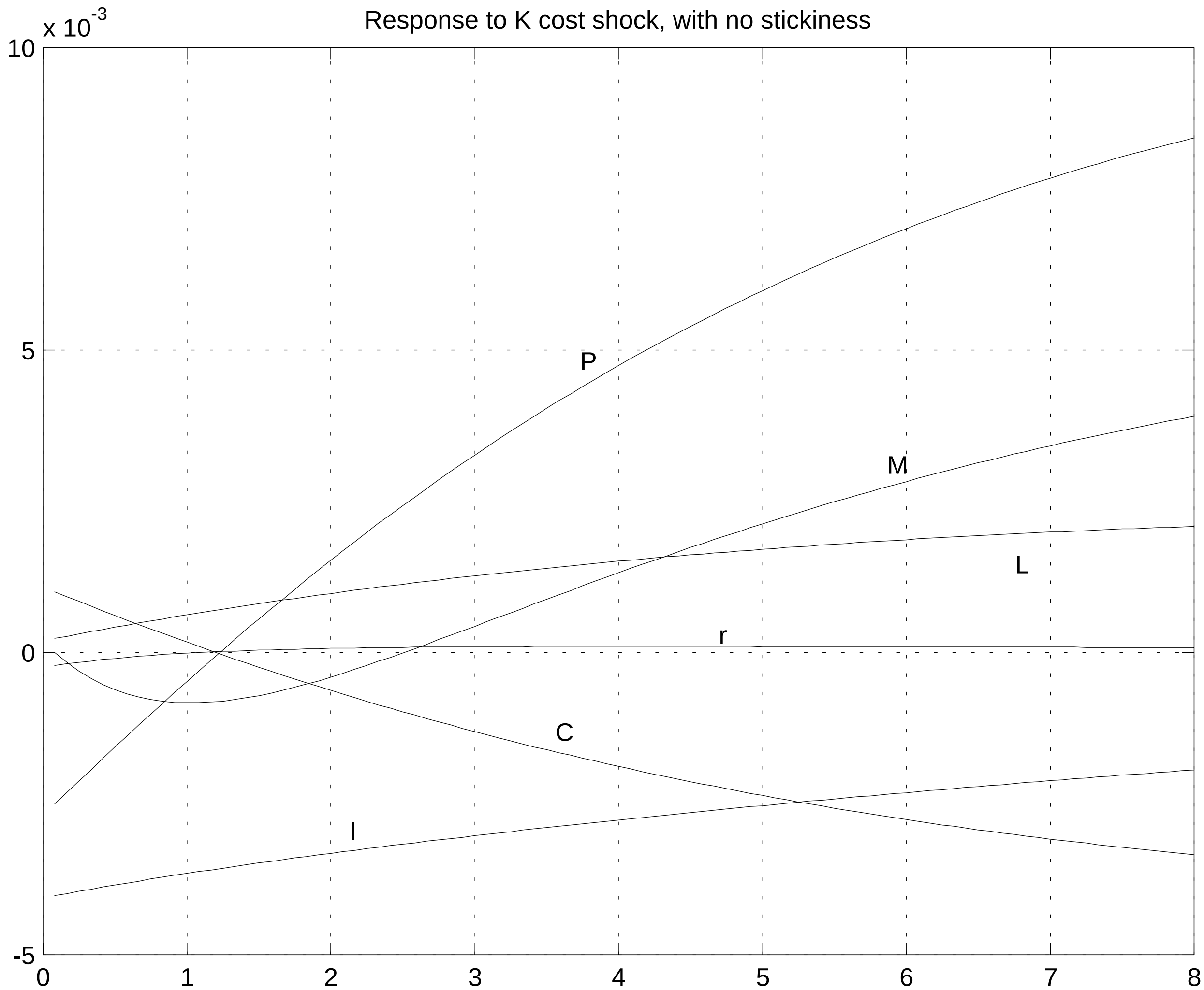


Response to M policy shock, with no stickiness, Taylor rule
logs, except r at annual rates and I scaled by I/C
years
r
I
P
M
C and L lie almost on x axis
2
0
-2
-4
-6
-8
-10
0
1
2
3
4
5
6
7
8

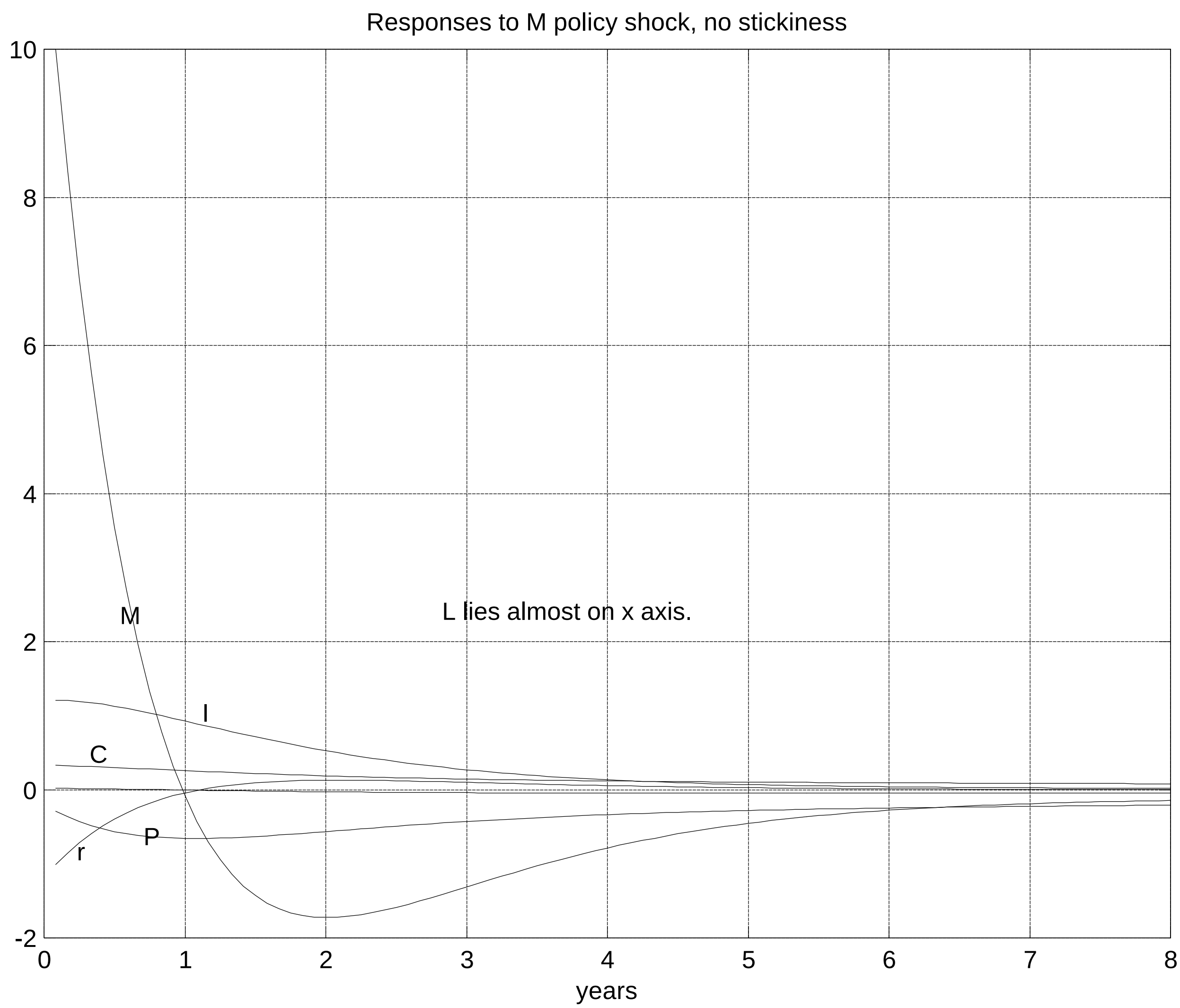

Response to M policy shock, with stickiness, Taylor rule (s.c.shock)
logs, except r at annual rates and I scaled by I/C
years
I
r
C
P
L lies almost on the axis
M
1
0
-1
-2
-3
-4
-5
-6
-7
0
1
2
3
4
5
6
7
8

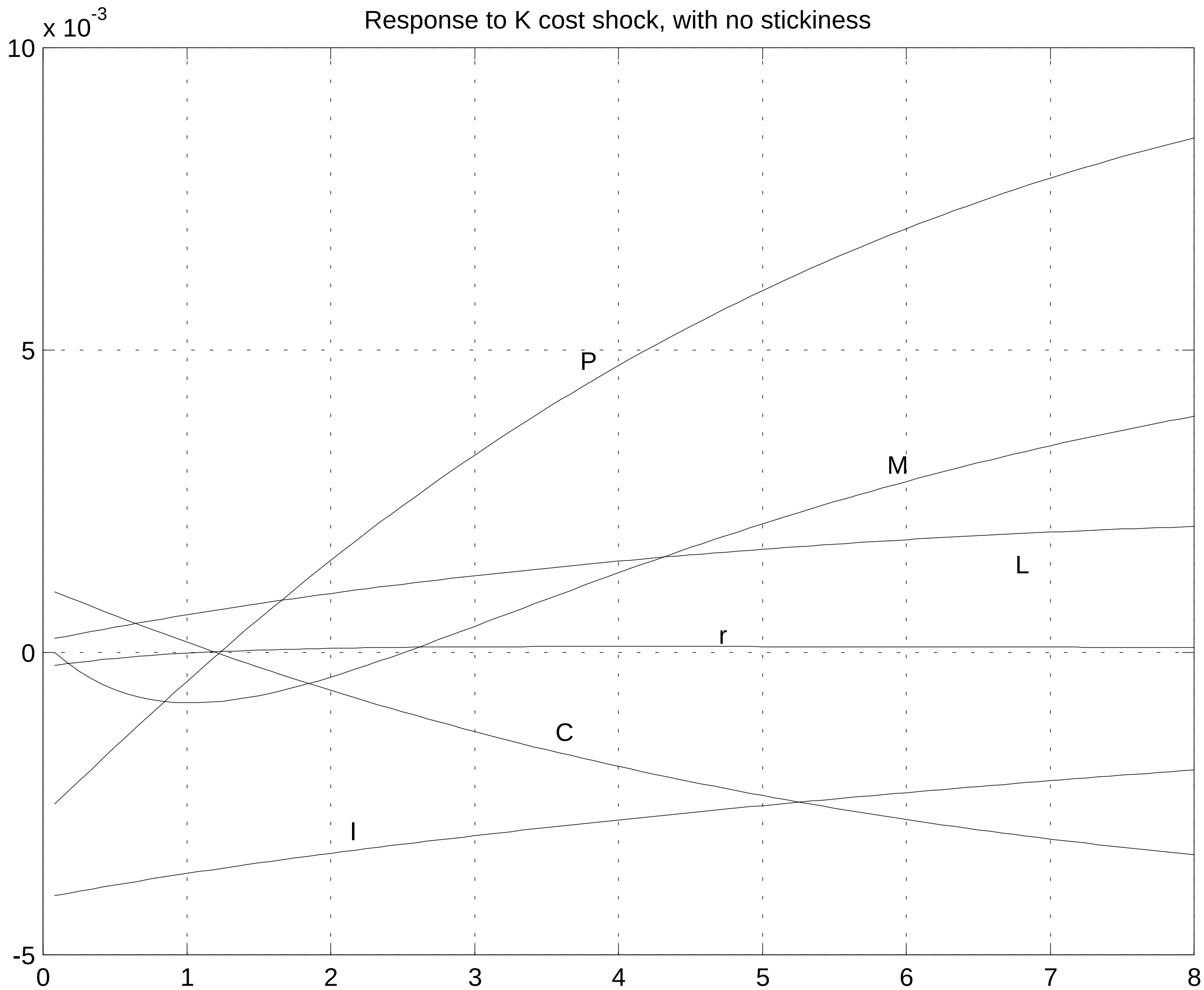


Response to K cost shock, with neutral stickiness
x 10^-3
8
6
4
2
0
-2
-4
-6
logs, except r at annual rates and I scaled by I/C
0
1
2
3
4
5
6
7
8
years
P
M
L
r
C
I



Response to K cost shock, with no stickiness
x 10^-3
10
5
0
-5
0
1
2
3
4
5
6
7
8
P
M
L
r
C
I


Responses to M policy shock, no stickiness
10
8
6
4
2
0
-2
0
1
2
3
4
5
6
7
8
years
M
I
C
P
r
L lies almost on x axis.


Response to K cost shock, with no stickiness
x 10^-3
10
5
0
-5
0
1
2
3
4
5
6
7
8
P
M
L
r
C
I