ACTIVE FISCAL, PASSIVE MONEY EQUILIBRIUM IN A PURELY BACKWARD-LOOKING MODEL

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ABSTRACT. The active money, passive fiscal policy equilibrium that the fiscal theory of the price level shows can deliver a uniquely determined stable price level is sometimes criticized as relying heavily on agents in the economy rationally forecasting the distant future. This type of equilibrium invokes a "transversality condition" on individual behavior, which is an implication of optimal behavior of agents that rationally forecast the distant future. The mechanism underlying the determination of the price level does not rely on perfect rationality, though. It is based on the simple idea that if agents rapidly accumulate wealth, they will eventually try to spend it. This note displays an old-fashioned Keynesian disequilibrium model, with wages and prices evolving slowly in response to excess demand and agents following a rule-of-thumb consumption function. In this model, the active fiscal, passive money policy combination can produce a stable equilibrium with determinate price level, so long as there is a wealth effect in the consumption function.

It is easy to understand how the government budget constraint becomes a source of potential instability if fiscal effort is unresponsive to the level of debt. If primary surpluses simply fluctuate around a stable mean, with no response to the level of debt, the government budget constraint becomes an apparently unstable difference equation, making real debt grow at the real interest rate. The fiscal theory of the price level brings out the fact that real debt generally nonetheless does remain stable under such a fiscal policy, because inflation and nominal interest rates react to stabilize debt. The causal chain leading to this result is that debt increasing at the real interest rate, while the time path of consumption remained stable, would be incompatible with individual optimization. Individuals who perceived themselves to be on such a trajectory would try to increase their spending, which would generate the inflation that maintained stable real debt. Under rational expectations, the ex ante real rate generally remains stable, so that the inflation that brings the real value of debt into line must occur as surprises.

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The formal, rational expectations models that lead to this conclusion embody this causal mechanism in the "transversality condition" of private agents. This condition's mathematical derivation in a particular model can be subtle. Formally, it often takes the form

$$E[\beta^t B_t \lambda_t / P_t] \xrightarrow[t \to \infty]{} 0$$
,

where B is nominal debt, λ_t is the marginal utility of consumption, P is the price level, and β is the discount factor in the representative agent's intertemporal utility function. On the face of it, assuming this condition characterizes individual behavior requires that the representative agent have a good understanding of calculus.

However, the behavioral mechanism at work here does not require full rationality. In fact, it is very similar to the informal intuition underlying the effect of the money stock on prices in a simple monetarist model. In both classes of models, the idea is that agents who found themselves with a more valuable stock of government paper than they have a use for, would try to get rid of it by spending more. This note illustrates this point by displaying a model with old-fashioned Keynesian "disequilibrium" price adjustment and purely backward-looking economic agents, in which a government policy of fixing the real primary surplus and the nominal interest rate at constant levels does not lead to explosive growth of debt, and in fact leads to a uniquely determined, stable price level. In this model the changes in prices and interest rates that keep real debt from exploding do not arrive as surprises; adjustment to shocks takes place over time and typically is oscillatory.

The model works by assuming consumers adjust the rate of growth of their spending up or down in proportion to the deviation of their holdings of real debt from some target level. Stability requires that the constant of proportionality be strong enough.

I. THE MODEL

Consumption function:

$$\dot{c} = \gamma_c (b - \bar{ au}/\bar{
ho}) + \varepsilon_c$$
 ,

where *c* is the log of consumption, *b* is the real value of government debt, $\bar{\tau}$ is the steady state level of lump sum taxation, and $\bar{\rho}$ is the steady state level of the nominal interest rate. ε_c is a disturbance to this equation that increases consumption. It can be thought of either as white noise (derivative of a Wiener process) or as randomly arriving zero-mean discrete "lumps" that produce discontinuities in the time path of *c*.

Government budget constraint:

$$\dot{b}=b
ho-\dot{p}b- au$$
 ,

where τ and ρ are the actual levels of lump sum taxation and the nominal interest rate and p is the log of the price level.

Fiscal policy:

$$\dot{ au} = \gamma_f(ar{ au} - au) + arepsilon_f$$
 ,

where ε_f is a disturbance to the rate of growth of τ . Fiscal policy is to put τ on an exogenous, stationary path. If ε_f stays at zero, τ is constant.

Monetary policy:

$$\dot{\rho} = \gamma_r(\bar{\rho} - \rho) + \varepsilon_m$$
,

where ε_m is a disturbance in the policy rule. Monetary policy is to put the nominal interest rate on a stationary exogenous path, and if there are no disturbances the nominal interest rate remains constant.

Production:

$$c = \alpha n$$
,

where n is the log of labor input and α is a Cobb-Douglas exponent. The model implies that agents spend all their income, both wage and "dividend" income, so as to make this social resource constraint hold identically.

Price adjustment:

$$\dot{p} = \gamma_p(w - p - \log(\alpha) + n * (1 - \alpha)) + \varepsilon_p$$

where w is the log of the nominal wage, and p is the log of the price level. So prices increase when the real wage exceeds the marginal product of labor.

Wage adjustment:

$$\dot{w} = \gamma_w n + \varepsilon_w$$
.

So the nominal wage rises when employment is above the normal level of 1.0, and falls when it is below.

Figure I shows the impulse responses of this system, linearized around its steady state with the parameters shown in Table 1. Because of the identity $c = \alpha n$, we can solve to eliminate *c* from the system, so that the "Consumption function" equation above has \dot{n} on the left, a speed of adjustment parameter γ_n instead of γ_c , and a shock ε_n rather than ε_c . It is this version of the model that is shown in the impulse responses.

Each shock is modeled as creating a discontinuous jump in the linear combination of variables that appears as a time derivative in the corresponding equation. In the absence of such disturbances, the variables appearing with dots must have continuous paths. For most equations, there is just one "left-hand side" variable that appears as a time derivative. However the government budget constraint contains two: b and \dot{p} . This implies that the price shock ε_p , which makes p jump discontinuously, must leave the level of nominal debt $(e^p \cdot b)$ unchanged, and thus that the

 $^{^{1}}$ Prices and wages contain a drifting component, so have no unique steady state value, even though the system implies a unique value for them at each date, given history and the steady state value of the real wage, w - n, is unique.

$$\gamma_n$$
 $\bar{\tau}$ $\bar{\rho}$ α γ_p γ_w γ_f γ_r 2.00 0.03 0.03 0.70 4.00 0.30 0.10 0.30 TABLE 1. Parameter values producing impulse responses of Figure I

jump in p must correspond to the same proportional, but opposite-signed, jump in

The impulse responses show that this model initially has the same qualitative behavior that arises in the simplest flex-price FTPL models. A positive shock to taxes τ reduces debt, inflation, and output. Unlike the simple flex-price models, though, this model implies the decline in debt reverses after 10 years, and the decline in inflation is persistent. Of course since this is a Keynesian model, the deflation is accompanied by depressed output, and since the model has no Fisher relation, deflation does not generate any movement in the nominal interest rate.

An upward policy shock to the nominal interest rate is initially expansionary, as in a simple rational expectations flex-price FTPL model. This shows that the expansionary character of an interest rate rise in this policy configuration does not come from the Fisher relation's forcing inflation to move in the same direction as the nominal interest rate, since in this model, there is no Fisher relation. The reason interest rate increases are expansionary both here and in RE FTPL models is that the rise in interest rates, under this fiscal policy, feeds through into increased rates of issuance of nominal debt. Because there is only a slow response of prices to the increased debt issue, real debt rises and induces increased consumption and employment. In a flexprice FTPL model the rise in interest rates would just raise the inflation rate, with no movement in real variables.

The parameter values that produced these impulse responses are shown in Table 1

Though the solution is stable with these parameter values, slowing the response of prices to the real wage can make the solution explosively oscillatory. For example, with the other parameters unchanged, reducing γ_p to 1 implies slowly explosive oscillations, while $\gamma_p=1.5$ still implies slowly damped oscillations. The period of the oscillations is about 8 years and is not very sensitive to γ_p in this range.

II. CONCLUSION

This model is not meant to be an improvement on rational expectations FTPL models, though they certainly have room for improvement. It is meant only as an example to demonstrate one point: that the existence of stable, unique equilibria under policies that peg the interest rate and leave fiscal effort unresponsive to the level of real debt does not rely on instant, far-sighted adjustments by rational agents. All that is required is a strong wealth effect on consumption and sufficiently rapid response of inflation to demand.

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