Pinning down the price level with the government balance sheet

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Controlling $M$

- Even leaving aside “transactions balances” that aren’t in anyone’s standard definition of $M$, the old story about how policy controls $M$ doesn’t work.

- The old story (simplified): Banks’ liabilities are deposits, $D$. They invest deposits in interest bearing loans, $L$. They are legally required to hold a proportion $\alpha$ of their deposits in non-interest-bearing “reserves”, $F$ — deposits with the Federal Reserve system and vault cash.

- Since reserves pay no interest, while loans do, banks keep reserves as close as possible to the legal minimum $\alpha D$. 
• The Federal Reserve system can control reserves $F$. $D = F/\alpha$, the “money multiplier”.

• $M = C + D$. $C$ not controlled, responds passively, but by controlling $D$, the Fed can control $M$. 
Currency drain in the Great Depression

- In the Depression, and in earlier contractions, public worries about the safety of bank deposits would lead to withdrawals from banks, increasing the ratio of $C$ to $D$.

- The Fed’s own balance sheet has currency and reserves ($C$ and $F$) as liabilities. If a “currency drain” occurs, $C$ increases and banks’ reserves shrink.

- Say $D$ declines by 100, $C$ increases by 100. Actual $F$ shrinks by 100, while required $F$ drops by $100\alpha$.

- If nothing else is done, banks must contract. They will try to sell assets or reduce loans, and deposit the proceeds with the Fed as reserves.
Countering a currency drain

- If total $M = C + D$ is to remain constant, the Fed must supply the now-missing reserves. Its total liabilities must increase by $(1 - \alpha) \cdot 100$, the banks’ gap between actual and required reserves.

- This requires the Fed to purchase assets to increase the size of its balance sheet to offset the contractionary effect of the currency drain. Otherwise, if $F + C$ rather than $F/\alpha + C$ remains constant, an increase in $C$ of 100 makes $M$ decrease by $100(1/\alpha - 1)$.
The Friedman-Schwartz critique

- Milton Friedman and Anna Schwartz used the $MV = PY$ framework and the money multiplier idea to argue that the Fed in the 1930’s failed to understand the need to expand its balance sheet in the face of banking system contraction, and that this led to the severity of the depression.

- In the recent crisis, there was no substantial currency drain and in the US banks subject to reserve requirements were not initially a main focus of financial stress. Nonetheless the Fed intervened, having learned the Friedman-Schwartz lesson, but wisely casting $MV = PY$ thinking aside.
What’s wrong with the old story?

- As part of the TARP legislation that in late 2008 authorized $700 billion in “bailout” funds, the Federal Reserve was given the right to pay interest on reserves, and it now does so.

- The approximate ratio of actual to required reserves today: 18.

- The interest rate on reserves: 0.25%. The interest rate on US 30-day Treasury bills: 0.01%.

- So the notion that banks should be eager to keep reserve balances close to the minimum required level no longer applies. There is no “money multiplier”.


Tax backing

- We are going to consider a very specific, simple model of price level determination.

- It can generate both the usual case of unbacked, “bubble” money with multiple equilibria and no uniquely determined price level, and that of “backed” money, in which even a small capacity to tax (more precisely, to tax in excess of current expenditures) implies a uniquely determined price level.

- Though the model is simple and abstract, the basic pattern of results applies in a wide variety of models with money.
Though in this perfect-foresight model each equilibrium is distinct and permanent, if we introduced randomness, there would be indeterminacy in prices every period.
People in the model

- Infinite sequence of two-period-lived agents, no population growth.
- Young endowed at birth with one unit of the single good.
- They can store it, in which case a fraction $\theta < 1$ of it survives to their second period of life, when they can eat it.
- They can also purchase from the current old generation nominal bonds, trading goods for bonds.
- At the initial date $t = 1$, there are old from generation 0 who have $B_1$ units of new one-period debt to sell to the young at $t = 1$. 
Government

- The bonds pay interest and are one-period bonds. The government exchanges each unit of mature bonds for $R$ units of newly issued one-period nominal bonds.

- The government may also impose lump sum taxes on the old, in which case the old must use some of their resources to pay the tax.
Variables

(All are per capita.)

$C_{1t}$: consumption while young (i.e. at time $t$) of the generation born at $t$

$C_{2,t+1}$: consumption while old (i.e. at time $t + 1$) of the generation born at $t$

$S_t$: amount put into storage at $t$ by the young at $t$

$B_t$: nominal bonds purchased by the young at $t$

$P_t$: price level at $t$. The rate at which new bonds trade for goods.

$R$: the gross nominal interest rate

$\tau$: lump sum tax on the old
Optimization problem of a typical agent

\[
\max_{C_{1t}, C_{2, t+1}, S_t, B_t} \{\log C_{1t} + \log C_{2, t+1}\}, \text{ subject to}
\]

\begin{align*}
\lambda_1 : & \quad C_{1t} + S_t + \frac{B_t}{P_t} = 1 \\
\lambda_2 : & \quad C_{2, t+1} = \frac{R B_t}{P_{t+1}} + \theta S_t - \tau \\
S_t & \geq 0, \quad B_t \geq 0.
\end{align*}
Government budget constraint, market clearing

\[ B_t = RB_{t-1} - P_t \tau \]

Market clearing imposed implicitly by using the same symbol, \( B_t \), for the bonds purchased by the young and the bonds returned (net of taxes) to old in exchange for maturing bonds.
First-order conditions

\[ \partial C_1 : \quad \frac{1}{C_{1t}} = \lambda_1 \]

\[ \partial C_2 : \quad \frac{1}{C_{2t}} = \lambda_2 \]

\[ \partial B : \quad \frac{\lambda_1}{P_t} = R \frac{\lambda_2}{P_{t+1}} \quad \text{if} \quad B_t > 0 \]

\[ \partial S : \quad \lambda_1 = \theta \lambda_2 \quad \text{if} \quad S_t > 0 . \]
Equilibrium conditions

\[ \theta = \frac{C_{2,t+1}}{C_{1,t}} \quad \text{if } S_t > 0 \]

\[ \frac{P_t R}{P_{t+1}} = \frac{C_{2,t+1}}{C_{1,t}} \quad \text{if } B_t > 0 \]

\[ C_{1,t} + C_{2,t} + S_t = 1 + \theta S_{t-1} \quad \text{SRC, from private and government constraints} \]
Equilibrium with no storage, $B_t > 0$

Define

$$\rho_t = \frac{C_{2,t+1}}{C_{1t}} = \frac{RP_t}{P_{t+1}}.$$  

$$C_{2,t+1} = \rho_t C_{1t} = \rho_t (1 - C_{1t}) - \tau$$

$$\therefore C_{1t} = \frac{1}{2} - \frac{\tau}{2\rho_t}.$$
No storage and $\tau = 0$

\[ C_{1t} = \frac{1}{2} = C_{2t}, \text{ all } t \]  \hspace{1cm} (1)

\[ \therefore \rho_t = \frac{RP_t}{P_{t+1}} = 1 \]  \hspace{1cm} (2)

\[ \frac{B_t}{P_t} = \frac{1}{2} \]  \hspace{1cm} (3)

\[ \therefore P_1 = 2B_1 \]  \hspace{1cm} (4)
\[ S_t > 0, B_t > 0, \tau = 0 \]

\[ \rho_t \equiv \theta = \frac{R P_t}{P_{t+1}} \]

\[ \therefore \frac{B_t}{P_t} = \theta \frac{B_{t-1}}{P_{t-1}} \text{ from GBC} \]

\[ \therefore \therefore S_t + \frac{B_t}{P_t} = \frac{1}{2}, \quad S_t \xrightarrow{t \to \infty} \frac{1}{2} \]

\[ P_1 = \text{ anything in } (2B_1, \infty] \]
No storage and $\tau > 0$

\[ C_{1t} = \frac{1}{2} - \frac{C_{1t}\tau}{2(1 - C_{1,t+1})} \]

Steady state: \[ C_{1t} \equiv \frac{3 + \tau - \sqrt{1 + 6\tau + \tau^2}}{4} \]

For $\tau$ small: \[ C_{1t} \approx \frac{1 - \tau}{2} \]
Non-steady-state equilibria for no storage, $\tau > 0$?
\[ \tau > 0 \text{ and } S_t > 0? \]

Because \( \tau > 0 \), \( C_{1t} < .5 \). Suppose we had \( S_t > 0 \) but it were known that \( S_{t+1} = 0 \). Since \( \tau > 0 \), \( C_{1,t+1} < .5 \), but at \( t + 1 \),

\[ C_{2,t+1} > 1 + \theta S_t - C_{1,t+1} > .5. \]

But this is impossible if \( \rho_t < 1 \). So if \( S_t > 0 \), it must be positive at all subsequent dates also. But then the GBC is

\[ \frac{B_t}{P_t} = \theta \frac{B_{t-1}}{P_{t-1}} - \tau \]

which is a stable difference equation that converges to a negative value, which is impossible.