CLARIFICATION OF MONEY MULTIPLIER NOTES

The first set of notes had a section on the effects of a cash drain in which two bullet points read (expanded a bit for clarity) as follows:

> Say D (deposits) declines by 100, C (currency) increases by 100. Required reserves F drop by 100α. (Because a fraction $\alpha < 1$ of deposits must be held in reserve deposits at the Fed.) Thus if total $M = C + D$ is to remain constant, the Fed's total liabilities must increase by $(1-\alpha)100$. This requires the Fed to purchase assets to increase the size of its balance sheet to offset the contractionary effect of the currency drain. Otherwise, if $F + C$ rather than $F/\alpha + C$ remains constant, an increase in C of 100 decreases F by 100 and thereby decreases D by 100 and thereby makes M decrease by $100(1/\alpha - 1)$.

It may have been confusing that I began by saying C increased and D increased by the same amount, then shifted to a scenario where C increased but D decreased by more. If the Fed simply responds passively to the banks' need for currency, the banks will come to the Fed for 100 units of currency, converting 100 units of their reserve accounts into currency. The Fed's total liabilities are unchanged: C up by 100, F down by 100. But with F down by 100, reserve requirements mean that total D must shrink by $100/\alpha$, not just the initial 100. So the overall shrinkage in $M = C + D$ is $100(1/\alpha - 1)$, the decrease in D being only partially offset by the increase in C, as stated in the notes. In order to avoid this shrinkage, the Fed needs to increase reserves by enough so that $D - 100$ can be sustained as the new level of deposits. That is, it has to offset most of the initial decline of 100 in reserves, so that the total decline is only 100α, not 100.

The algebra may make this clearer. Let primed variables be values after the currency drain, unprimed the values before.

$$
\begin{aligned}
M &= C + D && \text{(definition of money)} \\
F &= \alpha D && \text{(reserve requirements)} \\
C' &= C + 100 \\
M' &= C' + D' \\
F' &= \alpha D'
\end{aligned}
$$

Taking C and C' as given by the public's demand for currency, this is five equations in six unknowns. What actually happens depends on what the Fed decides to do. One possibility is that it decides to act so that $M' = M$. The other is that it decides to act so that $F' + C' = F + C$, i.e. it simply provides currency to banks as they cash in their deposits with the Fed, and the Fed balance sheet does not increase or decrease. If $F' + C' = F + C$, The equations imply

$$M' - M = 100 + D' - D = 100 + (F - 100)/\alpha - F/\alpha = 100 - 100/\alpha\,.$$

In other words, this leads to a shrinkage of M of the size given in the notes. If instead the Fed acts so as to keep $M + D = M' + D'$, we get

$$M' = C + 100 + F'/\alpha = C + F/\alpha = M\,,$$

which in turn implies that $F - F' = 100\alpha$, and therefore that $C' + F' = C + 100 + F - 100\alpha$, so that the total balance sheet expands by $100(1 - \alpha)$. To make the balance sheet expand, the Fed usually undertakes open market operations, meaning they buy bonds, paying for the purchases with increases in reserve deposits. Many economists, notably Milton Friedman and Anna Schwartz in their classic *Monetary History of the United States*, argue that the Fed contributed to the Great Depression by failing to recognize the need to expand its balance sheet to fully offset currency drains.