Using Gensys

The program \texttt{gensys.m} is called from matlab by a statement of the form

\[
\begin{align*}
g1, c, impact, fmat, fwt, ywt, gev, eu &= \text{gensys}(g0, g1, c, psi, pi, div) \\
\end{align*}
\]

The first five arguments of \texttt{gensys} on the right-hand side of this expression correspond to the five coefficients in our canonical form for a rational expectations system:

\[
\Gamma_0 y(t) = \Gamma_1 y(t - 1) + c + \Psi z(t) + \Pi \eta(t),
\]

in which the \(z\)'s are exogenous random variables and \(\eta\) is a vector of endogenous disturbances satisfying \(E_t \eta_{t+1} = 0\). The last argument, \(\text{div}\), determines the size of root that is treated as “unstable” for determining existence and uniqueness. By default, if this argument is simply omitted, the program assumes that any root strictly greater than one must be suppressed in the solution, while roots that have absolute value one are retained. If you set \(\text{div}\) at, say, 1.05 instead, then roots less than 1.05 in absolute value are retained, those equal to or greater than 1.05 in absolute value are suppressed.

The output of the program determines the coefficients in a “solved” system of the form

\[
y(t) = \Theta y(t - 1) + \Phi z(t) + \sum_{s=0}^{\infty} AM^s BE_t z(t + s + 1).
\]

The matrices on the left-hand side of (1) correspond to those in (3) as follows:

\[
\Theta = G1, \quad \Phi = impact, \quad A = ywt, \quad M = fmat, \quad B = fwt.
\]

The returned value \(eu\) is a vector that contains 1 for true and 0 for false, with its first element characterizing existence and its second element characterizing uniqueness. So you hope for \(eu = [1; 1]\). The returned value \(gev\) is the generalized eigenvalue matrix. Its second column divided by its first column is the vector of eigenvalues of \(\Gamma_0^{-1} \Gamma_1\) if \(\Gamma_0\) is invertible, and its first column divided by its second column is the vector of eigenvalues of \(\Gamma_0 \Gamma_1^{-1}\) if \(\Gamma_1\) is invertible.

Note that in the exercise you are working on, the role of \(z(t)\) in (2) is played by a vector whose elements are \(df'_{t-1}\) and \(\epsilon_m(t-1)\). That is, \(z(t)\) is known at \(t - 1\), even though it is i.i.d. across time. Thus \(E_t z(t + 1) = z(t + 1)\), even though \(E_t z(t + s) = 0\) for \(s > 1\). So for this particular model two terms in \(z\) will enter on the right, giving us

\[
y(t) = \Theta y(t - 1) + \Phi z(t) + AB z(t + 1).
\]

The \texttt{gensys} program is available on my web page at

\[\text{http://www.econ.yale.edu/~sims/pub/gensys}\]

It calls two other programs, \texttt{qzdiv.m} and \texttt{qzswitch.m}, which are available at the same web address.