Econ. 511b Spring 1999 George Hall and Chris Sims

ISLM(?) Model Exercise

Consider the following "modern ISLM" model:

$$\text{IS:} \qquad y_t = \alpha E_t y_{t+1} - \phi \cdot (r_t - E_t \pi_{t+1}) \tag{1}$$

$$\text{Phillips Curve:} \qquad \pi_t = \gamma E_t \pi_{t+1} + \theta y_t \, . \tag{2}$$

At first we will consider monetary policy that sets a time path for r_t that does not react to π or y, so r_t can play the role of the "exogenous" z_{t+1} in the general setup we discussed in class. That general setup we write as

$$\Gamma_0 x_{t+1} = \Gamma_1 x_t + \Psi z_{t+1} + \Pi \eta_{t+1} \, , \tag{3}$$

in which we know $E_t \eta_{t+1} = 0$.

1. Put the system (1)-(2) into canonical form, showing how expressions in the parameters α, ϕ, γ, and θ fill the matrix coefficients in (3).
2. We verified in class that this system has a unique solution if all the eigenvalues of $A = \Gamma_0^{-1}\Gamma_1$ exceed one in absolute value. (Actually it is *only* in this case that there is a unique stable solution, but we did not really prove that in class.) Show that if $\gamma = \alpha = 1$, and $\phi > 0$, $\theta > 0$, one of the roots is always less than one.
3. Suppose that instead of setting r_t exogenously, monetary policy sets m_t, the log of nominal money balances, exogenously. Suppose also that there is a money demand equation of the form

   $$m_t = y_t + p_t - \xi r_t \, , \tag{4}$$

   where p_t, y_t and m_t are all in log units.

   Use the money demand equation to rewrite the system so that it involves just y, p, and m. You will have to use the fact that $\pi_t = p_t - p_{t-1}$. Check the eigenvalues of this system when $\gamma = \alpha = 1$ and $\xi = \theta = \phi = .5$. Because when written in terms of p rather than π the system involves an extra lag, you will probably have to expand the system to 3 dimensions to answer this question. I have not verified that the system is algebraically tractable, and it probably isn't. Use Gauss or Matlab on a computer to find A and its eigenvalues, unless you see a clever way to get an analytic result.