A Truly Keynesian Model: Discrete Time[1]

In this model, following Keynes, we postulate directly that prices do not adjust instantly to clear markets, without an explicit and complete story describing the individual behavior that leads to this result. Because markets do not clear, we cannot treat individual behavior as generated by optimization of choices of all quantities given all prices. Instead we take a "disequilibrium model" approach: we assume that certain quantities are taken by private agents as beyond their control, while postulating equations of adjustment for prices that are meant to reflect actions of agents attempting to improve their lot. In particular, we first describe a complete equilibrium model, then delete certain first order conditions implied by agent optimization, replacing them with price adjustment equations that depend on the degree to which the deleted first-order conditions are violated.

First we lay out the equilibrium model. It contains no capital – a critical omission for this kind of model if it is to give realistic behavior. However our objective here is to lay out the simplest framework in which we can observe the mechanism by which price rigidity can imply strong real effects of monetary or fiscal policy. The model also contains no money, only government debt. This is probably not so critical for getting the model to display realistic behavior, but has the disadvantage that it does not display the way a standard "LM" curve emerges in such a model.

I. The Market-Clearing Version of the Model

The model contains a large number of representative workers, and an equal number of representative firms. The workers own the firms, which return profits to the workers as dividends. There are government bonds that are held entirely by the workers. The workers' problem is

$$\max_{\{C_t,B_t,L_t\}} E\left[\sum_{t=0}^{\infty} C_t^{\phi}(1-L_t)^{\gamma}\beta^t\right] \tag{1}$$

subject to

$$C_t + \frac{B_t}{P_t} + \tau_t \le R_{t-1}\frac{B_{t-1}}{P_t} + \frac{W_t}{P_t}L_t + \pi_t \tag{2}$$

$$B_t \ge 0 \tag{3}$$

The first order conditions for this problem are (using the notation $U_t = C_t^{\phi}\cdot(1-L_t)^{\gamma}$)

∂C: $$\frac{\phi}{C_t} U_t = \lambda_t \tag{4}$$

∂L: $$\frac{\gamma}{1 - L_t} U_t = \lambda_t \frac{W_t}{P_t} \ . \tag{5}$$

∂B: $$\frac{\lambda_t}{P_t} - \mu_t = \beta R_t E_t \left[\frac{\lambda_{t+1}}{P_{t+1}} \right] . \tag{6}$$

From (4) and (5) (since, with no satiation, the budget constraint will always bind and keep $\lambda > 0$) we can obtain

$$\frac{\gamma C_t}{\phi \cdot (1 - L_t)} = \frac{W_t}{P_t} \ . \tag{7}$$

From (4) and (6), assuming the borrowing constraint (3) does not bind, we obtain

$$\frac{U_t}{P_t C_t} = \beta R_t E_t \left[\frac{U_{t+1}}{P_{t+1} C_{t+1}} \right] . \tag{8}$$

The firm has a non-dynamic optimization problem each period:

$$\max_{L_t} [\pi_t] \tag{9}$$

subject to

$$\pi_t \le A_t L_t^{\alpha} - \frac{W_t}{P_t} L_t \ . \tag{10}$$

The first-order conditions for this problem reduce (after eliminating the Lagrange multiplier) to

$$\alpha A_t L_t^{\alpha - 1} = \frac{W_t}{P_t} \tag{11}$$

The government budget constraint, in per capita terms, is

$$\frac{B_t}{P_t} + \tau_t = R_{t-1} \frac{B_{t-1}}{P_t} \ . \tag{12}$$

Adding the consumer and firm budget constraints (2) and (10), then subtracting the government budget constraint (12), gives us the social resource constraint

$$C_t = A_t L_t^{\alpha} \ . \tag{13}$$

To complete the model, we need to specify the government's interest rate and tax policies. Suppose these policies set

$$R_t \equiv \bar{R} \cdot \varepsilon_t^R \ . \tag{14}$$

$$\tau_t \equiv \bar{\tau} \cdot \varepsilon_t^{\tau} \ . \tag{15}$$

II. The Keynesian Version

In the Keynesian version, we treat individuals and firms both as unable to directly control *L*. In the short run, workers assume they must work the hours they are assigned, or remain out of work if no job is offered them. Firms assume they must, because there are no inventories, match production to the amount demanded at the going price, and that this entails their hiring exactly enough labor to produce this amount. We therefore delete from the conditions of equilibrium equations (5) (and therefore also (7)) and (11). They are replaced by a Phillips Curve equation

$$\frac{W_t}{W_{t-1}} = \left(\frac{P_{t-1}\gamma C_{t-1}}{W_{t-1}\phi\cdot(1-L_{t-1})}\right)^{\theta_W} \varepsilon_t^W \tag{16}$$

and a markup pricing equation

$$\frac{P_t}{P_{t-1}} = \left(\frac{W_{t-1}}{P_{t-1}\alpha A_{t-1}L_{t-1}^{\alpha-1}}\right)^{\theta_P} \varepsilon_t^P \ . \tag{17}$$

These equations use the ratios of the left- and right-hand sides of (7) and (11) as indicators of worker and firm discontent with the current price-quantity combination, and then postulate that nominal wages move so as to reduce worker discontent, while prices move so as to reduce firm discontent.

Assembling this into a single system, we can use (8), (12), (13), (14), (15), (16) and (17) to determine *P*, *L*, *W*, *C*, τ, *R*, and *B*. The system is close to log-linear, but not quite, because of the government budget constraint (12) and the appearance of $1-L_t$ as a factor in some places. In log-linearized form it is

$$(\phi-1)dc_t - \gamma\frac{L}{1-L}d\ell_t - dp_t + \eta_{t+1} = (\phi-1)dc_{t+1} - \gamma\frac{L}{1-L}d\ell_{t+1} - dp_{t+1} + dr_t \tag{8*}$$

$$bdb_t + \tau\cdot d\tau_t = \frac{R\cdot B}{P}\cdot\left(dr_{t-1} + db_{t-1} + dp_{t-1} - dp_t\right) \tag{12*}$$

$$dc_t = da_t + \alpha\cdot d\ell_t \tag{13*}$$

$$dr_t = d\varepsilon_t^R \tag{14*}$$

$$d\tau_t = d\varepsilon_t^\tau \tag{15*}$$

$$dw_t = (1-\theta_W)dw_{t-1} + \theta_W\cdot\left(dp_{t-1} + dc_{t-1} + \frac{L}{1-L}d\ell_{t-1}\right) + d\varepsilon_t^W \tag{16*}$$

$$dp_t = (1-\theta_P)dp_{t-1} + \theta_P\cdot\left(dw_{t-1} - da_{t-1} - (\alpha-1)d\ell_{t-1}\right) + d\varepsilon_t^P \ , \tag{17*}$$

where we have used lower case letters to indicate logs, except that *b* is used for the log of B/P and (inconsistently) τ is used for the log of τ as well as for τ itself. Unsubscripted variables refer to values at the deterministic steady state, around which we have linearized.

This system is not as big as it seems at first glance, because (14)* and (15)* assert that we can treat r and τ as exogenous error terms, and (13) can be used easily to eliminate c from the system. The resulting 4-dimensional system has coefficient matrices, in the notation of the linear rational expectations systems notes,

$$\Gamma_0 = \begin{bmatrix} \gamma X + (1-\phi)\alpha & 1 & 0 & 0 \\ 0 & 1 & 0 & R^{-1} \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \; \Gamma_1 = \begin{bmatrix} \gamma X + (1-\phi)\alpha & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ \theta_W(X+\alpha) & \theta_W & 1-\theta_W & 0 \\ -\theta_P(\alpha-1) & 1-\theta_P & \theta_P & 0 \end{bmatrix}, \tag{18}$$

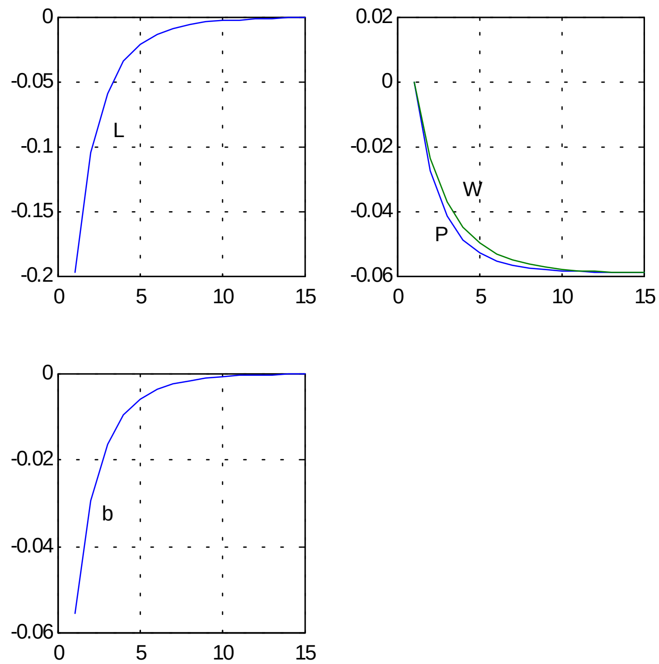
$$\Pi = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \; \Psi = \begin{bmatrix} \phi-1 & 1-\phi & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\beta^{-1}+1 & 0 & 0 \\ 0 & \theta_W & 0 & 0 & 1 & 0 \\ 0 & \theta_P & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{19}$$

The equations in this system correspond to (8), (12), (16) and (17), in that order. The variables are ordered as ℓ, p, w, b. In (18) we have used the notation $X = L/1-L$. In (19). the columns of Ψ correspond to a, lagged a, lagged r, τ, ε^W, and ε^P, respectively. The appearance of lagged shocks in the system, which are of course predictable in advance, means that to trace their effects we would have to use the full forward solution of the model, or else expand its dimension to create dummy variables to represent the lagged shocks. However we are interested primarily in examining the effects of shocks to τ, which appears only contemporaneously, for which the simple backward solution suffices.

With $\alpha = .3$, $\beta = .95$, $\gamma = \phi = .5$, $\theta_W = \theta_P = .2$, $\overline{R} = \beta^{-1}$, this system has one root of β^{-1}, one unit root, and two real roots of .45 and .69. Responses to a unit increase in τ, lasting only one year, are shown in the graph below.

Figure 1

Responses in log units to increase of 1 in log(τ)



Note that the fiscal contraction lowers labor input and causes deflation, as expected in a Keynesian model. But here, unlike in a conventional Keynesian model, we can see how this occurs via shifts in demand between goods and bond markets. The initial τ increase lowers b, because it lowers nominal debt B and initially P does not move. However future real taxes are back at their old steady state level, so the expected future real discount rates must rise to allow the unchanged level of future τ 's to match the lower initial b. The marginal utility of consumption is decreasing in L, once we take account of the fact that $C = L^{\alpha}$, so that periods of expected increase in L are periods of high discount rates (low discount factors). The initial drop in L makes it possible for the expected rate of change in L to be positive over most of the future, and hence for the discount rate to be higher over most of the future. The drop in L, via the wage adjustment equation, makes W begin to drop. Though b returns to its original equilibrium level, W and P do not. This means that B must eventually drop in proportion to P and W.

We can think of the impact of the tax increase on an individual as creating a budget problem: the tax has decreased the individual's wealth, so he will reduce current consumption and plan to rebuild wealth. This lowers demand for goods and directly reduces employment.

While the *ad hoc* price dynamics in this simple model are not common in the recent literature on sticky prices, the mechanism by which they allow fiscal policy to have real effects is basically the same as the mechanism in the recent sticky price literature: fiscal and monetary policy affect employment and output by influencing individuals' balance sheets, which in turn leads to shifts in demand between government paper liabilities and produced goods.

III. Sketch of an Exercise

An interesting question to explore is: Does the effect of the tax increase on employment get smaller if we "decrease stickiness" by making θ_W and θ_P larger? The answer in this and most similar models is that more rapid price adjustment does make the return of L to its steady state value quicker after a τ shock, but it does not have much effect on the initial response. You could fiddle with this yourself using Matlab and `gensys`. However under exam conditions you would not be asked to do so much arithmetic or algebra. You might, though, be given a partial solution of the model and asked to talk about it. For example, here you might be given the fact that with one setting of parameters (not exactly that corresponding to the graphs above) the linearized system, after multiplying through by Γ_0^{-1}, becomes

$$dy_t = Ady_{t-1} + Bdz_t + C\eta_t \,, \tag{20}$$

with $dy_t = \left[d\ell_t, dp_t, dw_t, db_t\right]'$, the column of B corresponding to $d\tau$ given by $\left[0,0,1,0\right]'$ and C given by $\left[-3.333,0,0,0,\right]'$. If you are then told further that A has the Jordan decomposition $A = VDV^{-1}$, with V given by

```
      0            -0.0000            -0.8222 + 0.2612i  -0.8222 - 0.2612i
      0             0.7071             0.2467 - 0.0784i   0.2467 + 0.0784i
      0             0.7071             0.2622 + 0.2554i   0.2622 - 0.2554i
 1.0000            -0.0000            -0.2179 + 0.0852i  -0.2179 - 0.0852i
```

V^{-1} given by

```
-0.2659             0.0471 - 0.0000i  -0.0471 + 0.0000i   1.0000
 0.4243             1.4142 - 0.0000i   0.0000 + 0.0000i        0
-0.5993 - 0.0279i  -0.4690 + 1.4762i   0.4690 - 1.4762i        0
-0.5993 + 0.0279i  -0.4690 - 1.4762i   0.4690 + 1.4762i        0
```

and D diagonal with 1.0526 , 1.0000, 0.7333 - 0.1247i , 0.7333 + 0.1247i on the diagonal, you should be able to use this information to figure out which linear combination of the data vector has to be constant – or equivalently, what is the linear relation determining η_t as a function of z_t, the exogenous shock vector. From that, you could determine how a shock in τ impacts current labor input. To turn this kind of analysis into a question about how θ_W and θ_P affect the response of ℓ to τ shocks would require that you examine two sets of matrices like this corresponding to different θ_W and θ_P, or else that the matrices be given initially in algebraic form so that you could see the dependence on θ_W and θ_P.