## Exercise Due 4/15

1. In the model with money that we have been discussing in class, consider whether there is a unique equilibrium price level under some variants of the transactions cost function $f$ that we have not considered in class. In particular, consider $f(v)=f_{N}(v)=v^{2}$, and $f(v)=f_{X}(v)=v^{2} /\left(1+v^{2}\right)$, under the policy of setting $M$ equal to a constant and using a compatible fiscal rule that maintains stability of the amount of outstanding real debt. Here the $N$ and $X$ subscripts on $f$ correspond to "normal" and "exotic". The former should require only a minor variation on arguments used in "A Simple Model..." and in class, while the latter raises complicated new issues, starting with the possibility of multiple steady-state $\bar{v}$ 's. In cases where there is no unique equilibrium price level, say what you can about the nature of the (multiple) possible equilibrium paths of prices. Consider whether, or to what extent, the non-uniqueness is resolved under a policy of pegging $\rho_{t}$ (the nominal gross interest rate) and $\tau_{t} / Y_{t}$ (the ratio of the real primary surplus to output) at constant values instead of holding $M$ fixed.
2. Suppose the government faces a real cost of providing real balances. In particular suppose that it is a constant fraction $\psi$ of the amount of real balances outstanding, so that the government budget constraint becomes

$$
\begin{equation*}
\frac{B_{t}+M_{t}}{P_{t}}=\rho_{t-1} \frac{B_{t-1}}{P_{t}}+\frac{M_{t-1}}{P_{t}}+\psi \frac{M_{t}}{P_{t}}+\tau_{t} \tag{1}
\end{equation*}
$$

No other aspect of the model specification changes, though when combined with the original private sector budget constraint, [1] implies a different social resource constraint. Now, in contrast to the original specification, there is an optimal rate of monetary growth $\pi$ when monetary policy sets $M_{t}=\bar{M} \pi^{t}$. Find how the optimal $\pi$ depends on the discount factor and $\gamma$.

