Econ. 511b Spring 1997 C. Sims

Dynamic Modeling Exercise

1. Consider the continuous time permanent income model in which an individual maximizes

$$\int_0^\infty \frac{C_t^{1-g}}{1-g} e^{-bt} dt \tag{1}$$

with respect to the time paths of C and W subject to

$$\dot{W} = rW + Y - C \ . \tag{2}$$

Log-linearize this model about steady state (except retain levels rather than logs for r), assuming that both Y and r are exogenously time-varying about their steady states. Find the time paths of C and W assuming $W_0 = 1$, $b = .05$, $g = 2$ for the following two cases:

i) $Y_t \equiv 1$, $r_t = .05 + .05e^{-.1t}$.

ii) $Y_t = 1 + .1e^{-.1t}$, $r_t \equiv 1$.

Note that there is not a unique steady state for this model. The steady state to which the system converges depends on the initial wealth. So part of the problem here is to determine what steady state the system converges to.

2. Consider the neoclassical growth model without adjustment costs, in which the representative agent maximizes (1) with respect to the time paths of C and K subject to

$$\dot{K}_t = A_t K_t^a - dK_t - C_t \ . \tag{3}$$

Log-linearize this model about steady state, assuming that A is exogenously time-varying, and find the time paths of C, K and $r = aAK^{a-1} - d$ when $a = .3$, $b = .05$, $K_0 = 1$, $\log A_t = 1 + .05 \cdot e^{-.1t}$.

3. Consider an economy in which there are two types, $i = 1,2$, of agents. Each receives a stream of income $Y_i(t)$, and the two types make loans to each other with a continuously adjusting instantaneous interest rate. They share the same discount rate but differ in their degrees of risk aversion. That is, the typical agent maximizes with respect to the *C* and *B* time paths

$$\int_0^\infty \frac{C_i(t)^{1-g_i}}{1-g_i} e^{-bt} dt \tag{4}$$

subject to

$$\dot{B}_i(t) = Y_i(t) + r(t)B_i(t) - C_i(t) \ . \tag{5}$$

Market clearing requires

$$B_1(t) + B_2(t) = 0 \tag{6}$$

(because we assume equal numbers of the two types of agents, and that agents issue bonds to each other – there is no external source of bonds and no government bonds).

Unlike the first problem, in which *r* is exogenous to the model, here *r* is determined within the model. It is still taken as exogenous by the agents – that is, they take its time path as given in solving their optimization problems – but the model has to determine $C_1, C_2, B_1 = -B_2$, and *r* jointly in one set of equations.

For $g_1 = 2$, $g_2 = 6$, $b = .05$, $Y_1(t) = 1 + .5e^{-.05t}$, $Y_2(t) \equiv 1$, $B_1(0) = 0$, log linearize (except retain levels rather than logs for **r**) around the steady state and find the time paths of C_1, C_2, B_1 and *r*. Here again, the steady state is not unique, but depends on $B_1(0)$. For this problem you may want to use `gensysct` .