

Exercise on Price Level Determination

Consider an economy in which individuals maximize

$$\int_0^{\infty} \frac{C_t^{1-g}}{1-g} e^{-bt} dt \quad (1)$$

subject to

$$C_t + \frac{\dot{B}_t}{P_t} = \frac{r_t B_t}{P_t} + Y_t - t_t . \quad (2)$$

Individuals take the time paths of P , r , and Y as beyond their control, and they choose C and B . The government has a budget constraint written in per capita terms (since there is constant population) as

$$\dot{B}_t = r_t B_t - P_t t_t . \quad (3)$$

Per capita endowment income Y evolves exogenously. The model then contains 5 endogenous variables, C , B , r , P , and t . The government can set three of these, subject to its constraint, so it has two dimensions of choice. The consumer, as we have already said, can set two of these, subject to its constraint, and thus has one dimension of choice. Usually the government is thought of as setting r and t , with its constraint determining B , but it can be thought of as choosing any two of the variables that appear in its constraint. (Of course, some policy choices may prove to be impossible – inconsistent with equilibrium.) In answering the questions below, first see if it is possible to proceed with the full nonlinear system (perhaps becomes it reduces to a linear one exactly). If necessary, linearize around a steady state.

1) Assume $Y_t \equiv \bar{Y}$, $r_t \equiv \bar{r} > b$. At time $t=0$, the amount of outstanding nominal debt is given as $B_0=2$. Before time 0, the economy has had $t_t \equiv \bar{t}$ and expected this level of primary surplus to continue forever. At $t=0$, it is announced, to the surprise of all, that henceforth the path of t will be $t_t = \bar{t}e^{-1t}$. Describe the time path, from before time 0 to infinity, of prices P , nominal debt B , real debt $b = B/P$, the inflation rate \dot{P}/P , and the conventional deficit inclusive of interest payments on the debt (i.e., \dot{B}). This solution should show prices that are in some sense “explosive”. Discuss why such explosive prices remain consistent with feasibility and optimality of individuals’ choices. If it is more convenient for you, assume numerical values $b=.05$, $r=.1$, $\bar{Y}=1$, $\bar{t}=1$.

2) Consider the situation of problem (1), but now with t set at .1 forever, with no surprise from this quarter at $t=0$. Instead, there is a surprise in Y , with Y following the path, from time $t=0$ onward, $Y_t = \bar{Y}e^{-1t}$. Again determine the time paths of the same set of variables you discussed in problem (1).

3) Suppose that instead of fixing r , the government fixes nominal debt B – i.e. runs conventional deficits of zero at all dates. With real primary surplus t also fixed, is there a unique equilibrium price level?

4) Suppose the government fixes nominal debt B as in (3), but instead of also fixing t , it announces a policy rule, believed by all, setting $t = f_0 + f_1 b$. Is there a unique equilibrium price level?

5) Prove that if the government sets $r = q_0 + q_1 P$ at all dates, and also sets $t \equiv \bar{t}$, there is in general no equilibrium. What about the case where $t \equiv \bar{t}$ is replaced by $\dot{B} = 0$?

6) Suppose the government sets $r = q_0 + q_1 P$, $t = f_0 + f_1 b$, with $q_1 > 0$, $f_1 > b$. Find the steady state values of all variables. Is there a unique equilibrium? Find the time paths of all the variables in the system for the case where $q_0 = .03$, $q_1 = .05$, $f_0 = -.1$, $f_1 = 1$, and the other parameters as we have assumed them in steady state above, supposing that at $t=0$, there is a surprise increase in q_0 from .03 to .05. Find the time paths of all the variables.