Econ. 511b Spring 1997 C. Sims

Permanent Income Exercise

Corrected 1:40PM 4/9. Formulas (1), (3) and (4) have been corrected to use discrete, rather than exponential discounting. Equation (5) has been corrected to make lagged, not current, Y appear on the right.

1. Consider an agent who values wealth not only for the consumption its income allows, but also directly – being rich provides utility even when consumption is not high because of the wealth. We model this with the following quadratic utility function.

$$E\left[\sum_{t=0}^{\infty}\left(C_t - \tfrac{1}{2}C_t^2 + q\left(W_t - \tfrac{1}{2}W_t^2\right)\right)b^t\right] \tag{1}$$

The constraint is as usual

$$W_t = (1+r)\left(W_{t-1} - C_{t-1}\right) + Y_t \ . \tag{2}$$

We assume Y is i.i.d. with mean $\bar{Y}$. The interest rate r is constant. The solution is constrained to have

$$E\left[b^{t/2}W_t\right] \to 0 \text{ as } t \to \infty \ . \tag{3}$$

Assume $(1+r)b = 1$ and find the value function and the policy function. Is consumption a random walk? Discuss the difference between this solution and that for $q = 0$ that we discussed in class. If we replace (3) with a requirement that $W_t \geq 0$, all t, can we still argue as in class that the solution under (3) cannot be a solution with the $W_t \geq 0$ condition? (A complete answer to this last question is probably hard, and depends on material that should presented Monday 4/14 in class.)

2. Consider the standard permanent income problem, now with serially correlated income. The objective function is

$$E\left[\sum_{t=0}^{\infty}\left(C_t - \tfrac{1}{2}C_t^2\right)b^t\right] \tag{4}$$

and the constraint is still (2) and (3). But now instead of i.i.d. Y, we have

$$Y_t = a \cdot \left(Y_{t-1} - \bar{Y}\right) + (1-a)\bar{Y} + e_t \tag{5}$$

where e is i.i.d. with zero mean. Find the value function and the optimal policy rule. Note that we are not here assuming $(1+r)b = 1$.