Permanent Income Exercise

Corrected 1:40PM 4/9. Formulas (1), (3) and (4) have been corrected to use discrete, rather than exponential discounting. Equation (5) has been corrected to make lagged, not current, Y appear on the right.

1. Consider an agent who values wealth not only for the consumption its income allows, but also directly – being rich provides utility even when consumption is not high because of the wealth. We model this with the following quadratic utility function.

\[
E \left[ \sum_{t=0}^{\infty} \left( C_t - \frac{1}{2} C_t^2 + \theta \left( W_t - \frac{1}{2} W_t^2 \right) \right) \beta^t \right]
\]

(1)

The constraint is as usual

\[
W_t = (1 + r)(W_{t-1} - C_{t-1}) + Y_t .
\]

(2)

We assume \( Y \) is i.i.d. with mean \( \bar{Y} \). The interest rate \( r \) is constant. The solution is constrained to have

\[
E \left[ \beta^{t/2} W_t \right] \to 0 \text{ as } t \to \infty .
\]

(3)

Assume \((1 + r)\beta = 1\) and find the value function and the policy function. Is consumption a random walk? Discuss the difference between this solution and that for \( q = 0 \) that we discussed in class. If we replace (3) with a requirement that \( W_t \geq 0 \), all \( t \), can we still argue as in class that the solution under (3) cannot be a solution with the \( W_t \geq 0 \) condition? (A complete answer to this last question is probably hard, and depends on material that should presented Monday 4/14 in class.)

2. Consider the standard permanent income problem, now with serially correlated income. The objective function is

\[
E \left[ \sum_{t=0}^{\infty} \left( C_t - \frac{1}{2} C_t^2 \right) \beta^t \right]
\]

(4)

and the constraint is still (2) and (3). But now instead of i.i.d.\( Y \), we have

\[
Y_t = \alpha \cdot (Y_{t-1} - \bar{Y}) + (1 - \alpha) \bar{Y} + \epsilon_t
\]

(5)

where \( \epsilon \) is i.i.d. with zero mean. Find the value function and the optimal policy rule. Note that we are not here assuming \((1 + r)\beta = 1\).