

## FISCAL THEORY IN AN OLD KEYNESIAN MODEL

### 1. MODEL DESCRIPTION

We consider an “Old Keynesian” model with FTPL. There is wage stickiness, but no attempt to model the microeconomic behavior that creates the stickiness. We simply postulate that nominal wages grow or decrease at a rate increasing in the discrepancy between the net marginal utility of work and the real wage. This not using the usual labor market clearing condition. The model has no money, but does have one-period nominal debt and lump-sum taxes. Here is its description.

**1.1. Households.** Note that households do not optimize with respect to labor,  $L$ . They work at the current wage in the amount that firms demand, and wages adjust in a direction that tends to make them happier.

$$\max_{C,B} \sum_{t=0}^{\infty} \beta^t (\log(C_t) + \log(1 - L_t)) \quad \text{subject to} \quad (1)$$

$$C_t + \frac{B_t}{P_t} = \frac{w_t}{P_t} L_t + z_t + R_{t-1} \frac{B_{t-1}}{P_t} - \tau_t \quad (2)$$

$C$  is consumption,  $L$  is labor,  $B$  is nominal bonds,  $P$  is the price level,  $R$  is the gross nominal interest rate,  $w$  is the nominal wage, and  $\tau$  is lump sum per capita taxes.  $z$  is dividends from the firms, which the households own.

Though it does not necessarily reflect only household behavior, we also list here the wage adjustment equation:

$$\frac{w_t}{w_{t-1}} = \left( \frac{C_{t-1}}{1 - L_{t-1}} \frac{P_{t-1}}{w_{t-1}} \right)^\delta + \varepsilon_w \quad (3)$$

**1.2. Firms.** Firms have no dynamic problem. They simply solve

$$\max_L z_t = e^{a_t} L_t^\alpha - \frac{w_t}{P_t} L_t. \quad (4)$$

$a$  is a technology shock.

**1.3. Government.** The government budget constraint is

$$\frac{B_t}{P_t} = R_{t-1} \frac{B_{t-1}}{P_t} - \tau_t \quad (5)$$

It sets interest rates and taxes (monetary and fiscal policy) by the rules

$$R_t = \left( \beta^{-1} \left( \frac{P_t}{P_{t-1}} \right)^\theta \right)^{1-\gamma} R_{t-1}^\gamma (1 + \varepsilon_M(t)) \quad (6)$$

$$\tau_t = -\phi_0 + \phi_1 \frac{B_{t-1}}{P_{t-1}} + \varepsilon_F(t) \quad (7)$$

**1.4. Technology.**

$$a_t = \rho a_{t-1} + \varepsilon_A(t) \quad (8)$$

The equations arising from this model, after apply household and firm FOC's, are listed in the `LandInflationModel.txt` file, which is both an input for the R programs below and a more or less readable list of R expressions for the system's equations. The expressions have no equal signs. They are “left-hand-side” minus “right-hand-side” expressions that should be zero when the equations are satisfied.

## 2. SETTING UP AND RUNNING CODE

This exercise will be *much* easier if you work in R, as it comes with R code that does most of the tedious programming and calculus for you. It is possible to do it in Matlab (or Octave, the open source alternative to Matlab). The provided R code does some of what the DYNARE system can do in a Matlab environment. If you already know DYNARE and Matlab, it might be almost as easy to do the problem in DYNARE as with the supplied R code. For Matlab without DYNARE, the `gensys.m` program is on the course site, but the work of `g0g1d`, `ssSolve`, `g0gleval`, and `impulsdtrf` you have to do yourself.

R can be downloaded for free from the internet. You will need to install the QZ package from the internet. This can be done with `install.packages('`QZ`')`, which should give you a menu of choices for the site to download from. You should also download the directories `csolve` and `gensys` from the links on the course web page. If you are in the directory containing these two subdirectories, from within R give the command

```
install.packages(c('`csolve`', '`gensys`'), repos=NULL)
```

Then you bring the package into your working space with `library('`csolve`')` and `library('`gensys`')`. In subsequent sessions on the same computer, you do not need to repeat the installation. The `library()` commands should be enough.

It is also possible to simply use `source()` to bring the programs in the `csolve/R` and `gensys/R` directories into your workspace. The advantage of doing a formal install of the two packages is that nicely formatted help files for them are available from within R if the packages are installed. You do have to install the QZ package in any case.

A script that loads a base version of the problem's model and plots impulse responses to the monetary policy shock is below. There are comments to explain what the code is doing. It assumes you have the model description file, `LandInflationModel.txt`, in the working directory. You can set your working directory with the `setwd()` command.

```
ex <- read.eqsys("LandInflationModel")
##
## ex is now an "eqsys" object, a vector of R expressions that define the
## equation system. They are all zero when the equations are satisfied.
## It has attributes vlist, param, and shock, each of is a list of the
## corresponding names that appear in the equation system. vlist are the
## variables being determined in the dynamic system, param are fixed
## parameters, and shock are the names of exogenous disturbances.
##
ex
## Typing ex and hitting return gives you a listing of the equation system
##
exd <- g0g1d(ex)
##
## exd is ex with an additional attribute: a collection of expressions that
## define the matrix of gradients of the equation system with respect to vlist.
##
x0 <- c(a=0, C=.7, L=.5, w=.7, P=1, B=10, R=1.05, tau=1)
##
## x0 is a guess at steady state values of the variables in the model. To
## be used as a starting point in solving for steady state. If you're not
## using this R code, you probably can more easily find the steady state
## analytically. Note that the names on components of x0 are essential, as
## are the names on the components of param.
##
```

```

param <- c(alpha=.7, beta=.95, rho=.9, theta=1.5, phi0=1, phi1=.1, delta=.3, gamma=.7)
##
## Sets a baseline vector of parameter values.
exss <- ssSolve(ex, x0, param, itmax=50)
##
## Finds a steady state. This model leaves the level of P and B
## indeterminate, but the numerical routine finds a steady state, even though
## it is not unique. The itmax parameter can be set higher if you have a
## system that's hard to solve or you have a bad initial guess x0.
##
exg0g1 <- g0g1eval(exd, x=exss$xss, param=param)
##
## Finds the matrices defining the local linearization of ex around the
## steady state. It uses "automatic differentiation" (not "symbolic
## differentiation").
##
exSoln <- with(exg0g1, gensys(g0, g1, psi=Psi, pi=Pi))
##
## Does the root-finding existence and uniqueness checking, etc. for the
## linear rational expectations model defined by exg0g1. The "with" function
## avoids having to write exg0g1$g0, exg0g1$g1, etc.
##
exSoln$eu
##
## We hope this displays "1 1", unless we are probing to see what parameter
## values imply non-existence or non-uniqueness.
##
exirf <- Re(impulsgensys(exSoln, 40))
##
## The impulse responses functions, as an nv x nshock x 40 array. The Re()
## strips off the (tiny, if the model is well specified) imaginary parts to
## avoid warning messages from the plot command.
##
plot(ts(t(exirf[ , "epsM", ]), main="Responses_to_epsM,_base_param", yax.flip=TRUE)
##
## plots responses to the monetary policy shock. It extracts the
## nv x 40 matrix of responses, transposes it, converts the matrix to a time
## series (ts) object, then plots it. The point of making it a ts object is
## that R knows to plot a ts matrix as a set of plots, stacked up, with a
## common x axis. The "yax.flip=TRUE" avoids having the axis labels
## partially overwrite each other.

```

### 3. THE OLD KEYNESIAN FTPL EXERCISE

- (1) Find the FOC's of the households and firms and show how they lead to the expressions in `LandInflationModel.txt` (Or else send me an email, promptly, explaining that you think there is an error.)

The FOC's, before eliminating Lagrange multipliers, are, for households,

$$\begin{aligned} \partial C : & \quad \frac{1}{C_t} = \lambda_t \\ \partial B : & \quad \frac{\lambda_t}{P_t} = \beta R_t E_t \left[ \frac{\lambda_{t+1}}{P_{t+1}} \right]. \end{aligned}$$

There is no  $\partial L$  FOC, because in this version of the model households take their labor as determined by employer's demand for labor at the current wage. However, for future reference in the flex-price version of the model below, we show here what the FOC for  $L$  would be if households saw  $L$  as a choice variable:

$$\partial w : \quad \frac{1}{1 - L_t} = \lambda_t \frac{w_t}{P_t}.$$

Firms have a static optimization problem:

$$\partial L : \quad \alpha e^{a_t} L_t^{\alpha-1} = \frac{w_t}{P_t}.$$

The rest of this question is just a matter of solving to eliminate Lagrange multipliers, identifying which equations are forward looking (and thus get a \* after their names in `LandInflationModel.txt`), and connecting notation of these equation to that of the text file.

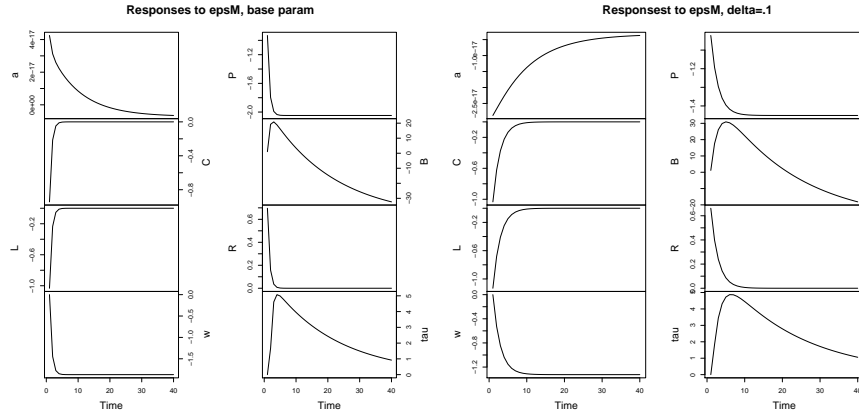
- (2) Construct impulse response plots for  $\varepsilon_M$  and  $\varepsilon_A$  shocks, for several values of the parameter  $\delta$ , which determines the degree of wage stickiness, but keeping the other elements of `param` at the values set in the script above. Does the model become more like a flex-price model when  $\delta$  gets larger?

The pattern is that i) if you make  $\delta$  very large (say 10), you get non-uniqueness; ii) when you have existence and uniqueness, the initial impact of a shock — either  $\varepsilon_A$  or  $\varepsilon_M$  — is about the same, regardless of the value of  $\delta$ ; iii) as  $\delta$  gets smaller, the effects of the shocks get more persistent; and iv) for large  $\delta$ , below the level that produces non-uniqueness, the responses are highly oscillatory.

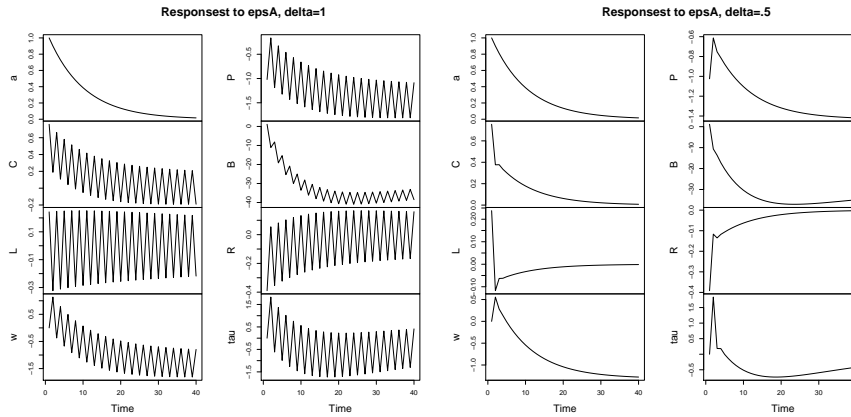
These results in part reflect the idiosyncracies of discrete time. When the  $\varepsilon_M$  shock raises the nominal rate,  $P$  drops, while  $w$  initially cannot respond, which raises the real wage and reduces employment and consumption. At low levels of  $\delta$ , the decline in prices persists, so expected inflation is negative, raising the real rate of interest and thus raising the expected growth rate of consumption. As delta increases, the rate of deflation in the initial periods, and thus the real rate, and thus the rate of growth of consumption, becomes larger, eventually to the point where consumption in the initial period grows so fast it overshoots steady state and produces oscillations.

In a continuous time version of this model, probably the oscillations do not occur. Instead the initial drop in consumption and  $L$  is greater with greater  $\delta$ , while the rate of return to the steady state is higher. (I haven't actually solved the continuous time version of this model, but this is what happens in similar models.)

Plots of a few of these impulse responses are below. Note that the  $a$  plot of course shows not reponse to  $\varepsilon_M$  shocks, which you can see by looking at the axis labels, despite there being a non-flat plotted

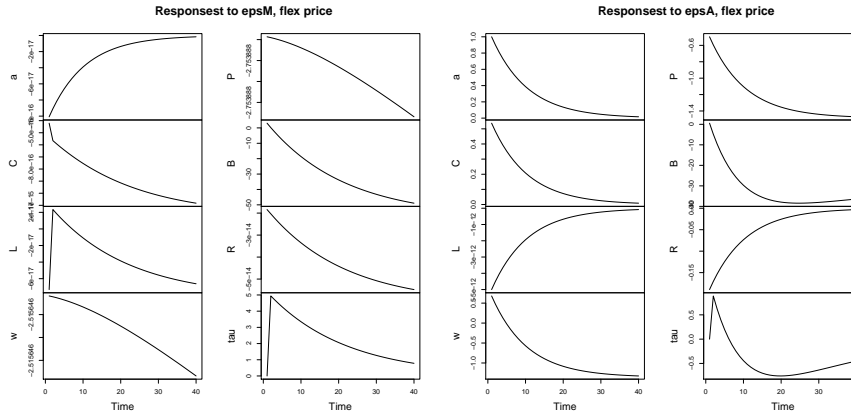


response.



- (3) Modify the `LandInflationModel.txt` file, replacing the wage adjustment equation with a standard labor market clearing equation, and repeat your impulse response calculations from above on this new model. This is to verify how a flex-price version of the model behaves.

With flexible prices,  $\varepsilon_M$  affects only nominal variables, with no reponse at all from  $C$ ,  $L$  or  $w/P$ . (You have to normalize the responses of  $w$  and  $P$  by their steady state values to see that their percentage responses to the shock are identical.)  $\varepsilon_A$  affects  $C$ , but has no effect on  $L$ , in contrast to the sticky-wage case where initial effects on  $L$  are large and invariant to  $\delta$ .



- (4) In Leeper's setup, there is a unique stable equilibrium when  $\theta > 1$  and  $\phi_1 > \beta^{-1} - 1$ , no stable equilibrium if  $\theta > 1$  and  $\phi_1 \leq 0$ , a unique stable equilibrium when  $\theta \leq 1$  and  $\phi_1 \leq 0$ , and indeterminacy if  $\theta \leq 1$  and  $\phi_1 > \beta^{-1} - 1$ . By varying the parameter values for this model and checking the  $e_u$  component of the solution, determine whether it appears that these same boundaries apply to this model.

The short answer is, yes the same boundaries apply. One starts to get some deviation from these boundaries in New Keynesian or Old Keynesian models when the monetary policy reaction function depends on output or some other measure of real activity, but in this simple model the results are exactly as in Leeper.

#### 4. POLICY GAME EXERCISE

In the standard natural rate Phillips curve setup, with government maximizing, by its choices of  $\pi$  and  $u$ ,

$$-\frac{1}{2} \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \theta u_t^2) \quad (9)$$

subject to the Phillips curve

$$u_t = -\alpha(\pi_t - E_{t-1}\pi_t + \bar{u} + \varepsilon_t), \quad (10)$$

display the full commitment optimum solution under rational expectations, and also the no-commitment myopic solution, in which the government simply minimizes current  $\pi_t^2 + \theta u_t^2$ . (These are in the notes, and other sources.)

Then consider instead the possibility that the private sector forms backward-looking expectations, with expected inflation next period equal to actual inflation this period. This implies the Phillips curve

$$u_t = -\alpha(\pi_t - \pi_{t-1}) + \bar{u} + \varepsilon_t. \quad (11)$$

Solve the optimal policy problem under this assumption. Compare the performance of the economy in this case to that with rational expectations. Does the solution emerge as self-confirming? That is, on the solution path is it in fact optimal to forecast inflation as equal to last period's inflation?

This problem can be done analytically, but it is a linear system, so you can solve it numerically instead, using reasonable parameter values and `gensys`. If you do it with `gensys`, add a dummy variable  $w_t = E_t \pi_{t+1}$  so as to have a distinct "forward-looking" equation.

The "temptation" to do a lot with inflation here is measured by  $\alpha^2 \theta$ . If this gets too large, the solution may not be stable. However for any given  $\alpha^2 \theta$ , there is a stable solution if  $\beta$  is close enough to (but below) one. With less discounting of the future, the unstable paths become unattractive. These results can be shown analytically, but they involve a lot of rather messy analysis of quadratic equations.

Below is the input file for the model with The expected next period inflation in the Phillips curve equal to current inflation:

```
Pcurve
u - (alpha * (pi - pil) + ubar + epsU)

FOC*
-pil - (alpha * theta * ul - beta * alpha * theta * u)

##-----

vlist
pi u

param
alpha beta ubar theta
```

shock  
epsU

With the parameters set as  $\alpha = -.5$ ,  $\beta = .9$ ,  $\theta = 1$ ,  $\bar{u} = 5$ , this gives us a steady state of  $\bar{u} = 5$ ,  $\bar{\pi} = .25$ . The immediate response to a unit shock in  $\varepsilon_U$  (from the `impact` element of the `gensys` output) is .35 and .83, respectively, in  $\pi$  and  $u$ . Unlike either the myopic or the commitment solution to the natural rate version of the model, though, the system is serially correlated here, so the variance is larger than the sum of squares of these two impact coefficients (which is .803). If you've studied some time series, you might know that the two variances can be calculated from the sum of squares of the two impulse responses to  $\varepsilon_U$ , which is .125 for  $\pi$  and .703 for  $u$ . adding up to .838. This plus the sum of squares of the means, is  $\bar{u}^2 + \bar{\pi}^2 + .838 = 25.9005$  as twice the per period loss function.

As shown in the lecture notes, the myopic solution for this model and these parameter values makes the steady state mean inflation rate 2.5 and the variances of inflation and unemployment .16 and .64, respectively, delivering a per period (times two) of 32.05, substantially worse than the adaptive expectation result, entirely because of the much larger steady state inflation rate.

The commitment solution make the mean inflation rate zero, and thus delivers twice the per period losses of 25.8, slightly better than what results with adaptive expectations.

The adaptive expectations solution here is not self-fulfilling: Though  $\pi$  is serially correlated, the serial correlation is weak. Forecasting  $\pi_t$  as  $\pi_{t-1}$  is worse than forecasting  $\pi_t$  as always equal to its mean of .25. The constant forecast gives a mean square error of .13, while the persistence forecast gives a mean square error of .21.

On an in-class exam you obviously can't do extensive computer calculations, and for this course you were not expected to know how to convert impulse responses into variances. But you should have been able to see the dominant effect on losses of the deviation of the mean inflation rate from its optimal steady state value of zero, and that in the adaptive expectations solution the outcome is serially correlated.