"MICRO-FOUNDED" STICKINESS

1. Dixit-Stiglitz monopolistic competition

Consumers' utilities depend on a consumption good aggregate

$$C_t = \left(\int_0^1 c_{jt}^{1-1/\theta} dj \right)^{\theta/(\theta-1)} ,$$

where j indexes a continuum of types of gooods and $\theta > 1$ determines how substitutable the goods are for one another. If good j has price j, consumers will maximize aggregate C subject to $\int p_j c_j \, dj = X$, where X is total expenditure on consumption goods. This implies that

$$C^{1/\theta} c_j^{-1/\theta} = \lambda p_j \text{ , all } j \, , \tag{$*$}$$

$$\int p_j c_j \, dj = C^{1/\theta} \cdot C^{(\theta-1)/\theta} / \lambda = C/\lambda = X \, . \tag{$**$}$$

2. Price Index

It is natural to think of the aggregate consumption price index as X/C, which makes it just λ in the above expression. It is not hard to show, using $(*)$ and $(**)$ that

$$X = PC = CP^{\theta} \int p_j^{1-\theta} dj$$

$$\therefore P = \left(\int p_j^{1-\theta} dj \right)^{1/(1-\theta)}$$

3. Firms

Everyone hires labor in the same market at the same wage. (Different, and simpler than, Woodford's "base" model.)

With no frictions:

$$\max_{p_j} \left\{ p_j q_j - w\alpha q_j = (p_j - w\alpha) C \left(\frac{p_j}{P} \right)^{-\theta} \right\} ,$$

where q_j is quantity sold, w is the wage, and α is unit labor requirements. The solution is

$$p_j = \frac{\theta}{\theta - 1} w\alpha \, .$$

That is, price is a fixed markup over labor costs. Usually the "markup" is defined as $(p_j - w\alpha)/p_j = 1/\theta$, i.e. the proportion of the price that is not absorbed in variable costs.

4. Monopolistic distortion

- We are considering a version of this model with a fixed range of goods j and linear production technology.
- In deterministic steady state, with fixed labor supply, the allocation is optimal.
- But if we allow labor/leisure choice, the monopolistically competitive equilibrium is not generally optimal.
- As usual, monopoly power restricts output and, therefore, employment.
- Most equilibrium models of this type assume a perfect subsidy to production, that makes the deterministic steady state output level optimal. (!)

5. Price adjustment friction

We have no nominal rigidity or money illusion yet. The Dixit-Stiglitz setup has just given us agents who set prices in an optimization problem. Possible devices:

- Prices can only be revised at fixed intervals. (Taylor, Stan Fischer, first case in Woodford book, "Tayor contracting")
- Firms can only revise prices at exogenously determined random times. ("Calvo pricing")
- Firms face costs of adjusting prices, with the costs increasing as the rate of change of price increases. ("Menu costs", quadratic adjustment costs)
- Firms face costs of adjusting prices, but choose the timing of price adjustments by comparing costs of price change to benefits of it. ("State-dependent pricing")
- Firms can only revise prices at fixed intervals, or pay a cost to increase the frequency of price revisions, but when they do revise, they set an optimal time path of prices, not a fixed price. (Mankiw-Reis inattention)
- Firms and/or consumers only loosely keep track of the determinants of optimal prices and quantities, because of (Shannon information theory based) costs of responding rapidly and accurately to information. (Sims "rational inattention")

6. Calvo pricing

A fixed, randomly selected, fraction δ of firms choose new prices each period. Since there are no costs associated with the size of the price change and all firms have the same demand and cost parameters, all the firms, no matter what their previous price, will choose the same current price p_t^*. The price dynamics will therefore be

$$\bar{P}_t = \delta p_t^* + (1-\delta)\bar{P}_{t-1} \quad \text{or, in continuous time}$$
$$\dot{\bar{P}}_t = \delta(p^* - \bar{P}_t)\,. \tag{$*$}$$

Note that the $\bar{P}_t$ appearing here is the arithmetic average of prices whereas the exact price aggregate is a different average, closer to a harmonic mean. The formulas above are approximately correct for the exact price aggregate when price differences across firms are small.

7. Price setter's problem

A firm choosing its price at t maximizes with respect to p_{jt}

$$E_t\left[\sum_0^\infty \beta^t(1-\delta)^t \frac{p_{jt}q_{j,t+s} - \alpha w_{t+s}q_{j,t+s}}{P_{t+s}}\right]\,.$$

Note that we have them maximizing real (aggregate price deflated) profits. They discount by the consumers' discount rate β — which in a general equilibrium with risk averse agents is justifiable only as an approximation when uncertainty is not too great — plus the rate at which commitment to this fixed price is expected to decay in the future. Future quantities sold will be determined as $C_{t+s}(p_{jt}/P_{t+s})^{-\theta}$, that is by future aggregate consumption and future aggregate prices relative to the fixed p_{jt}.

8. p^*

Letting $\phi = \beta(1-\delta)$,

$$p_t^* = \frac{E_t\sum_0^\infty \phi^s\theta\alpha w_{t+s}P_{t+s}^{\theta-1}C_{t+s}}{E_t\sum_0^\infty \phi^s(\theta-1)P_{t+s}^{\theta-1}C_{t+s}}$$

Log Linearization:

$$\tilde{p}_t^* = (1-\phi)E_t\sum_0^\infty \phi^s\tilde{w}_{t+s}$$

Here $\tilde{p}$ and $\tilde{w}$ are log deviations from steady state.

Substituting this in ($*$) gives us an equation determining the growth rate of the current price level as a function of the ratio of an average of expected future nominal wages to the current price level:

$$\tilde{p}_t = \delta(1-\phi)E_t\sum_0^\infty \phi^s\tilde{w}_{t+s} + (1-\delta)\tilde{p}_t\,.$$

9. The New Keynesian Phillips Curve

A Phillips Curve usually relates inflation to unemployment or some other real variable. To get an equation in this form, we write $W_t = w_t - p_t$ for the real wage and $\pi_t = p_t - p_{t-1}$ for inflation. We drop the "$\sim$" over every variable — all variables are log deviations from trend.

$$\begin{aligned}
\pi_t &= \delta(1-\phi)E_t \sum_0^\infty \phi^t W_{t+s} \\
&\qquad + \delta(1-\phi)E_t \sum_{s=0}^\infty \phi^s \left(\sum_{v=0}^s \pi_{t+v} + p_t \right) - \delta p_t \\
&= \delta(1-\phi)E_t \sum_{s=0}^\infty \phi^s W_{t+s} \\
&\qquad + \delta(1-\phi)E_t \left[\sum_{v=0}^\infty \pi_{t+v} \sum_{s=v}^\infty \phi^s \right] + \delta p_t - \delta p_t \\
&= \delta(1-\phi)E_t \sum_{s=0}^\infty \phi^s W_{t+s} + \delta E_t \sum_{v=0}^\infty \phi^v \pi_{t+v}
\end{aligned}$$

10. The familiar form

We can rewrite the last expression as

$$\pi_t = \delta(1-\phi)W_t + \delta\pi_t + \phi E_t \pi_{t+1}$$

and thus

$$\pi_t = \frac{\delta(1-\phi)}{1-\delta} W_t + \beta E_t \pi_{t+1} \,.$$

This last is the way the New Keynesian Phillips Curve is usually written: Inflation is a coefficient times a measure of real cost pressure (here the real wage) plus the discount factor times expected inflation next period.

11. Welfare costs of inflation

- Our stripped-down setup makes labor requirements for good j linear in c_j.
- Total labor is therefore proportional to $\int c_j \, dj$.
- Utility depends on

$$C = \left(\int c_j^{1-\frac{1}{\theta}} \, d\theta \right)^{\frac{\theta}{\theta-1}} \,,$$

  which is a concave function of $\{c_j\}$ symmetric in its arguments.

- Therefore at a given level of total labor input, utility is maximized when c_j is constant across j.

12. Welfare costs, II

- When inflation varies, or even if it is constant and non-zero, Prices, and hence outputs, of goods whose prices have been set at different times will differ.
- A Taylor expansion of the consumption aggregate shows that with p_j i.i.d. across j, consumption decreases in proportion to the variance of p_j for small variances.
- Taken literally, the model implies even steady, predictable, inflation produces welfare losses, because it too will result in output levels varying across firms.
- Most users of this setup do not take this latter conclusion as realistic.

13. Welfare costs, Calvo vs. other types of stickiness

- State-dependent menu cost pricing implies the firms that adjust will be those with p_t^*/P_t most out of alignment.
- This limits the range of deviation of outputs from optimal levels, implies smaller welfare costs – even zero welfare costs in some special cases.
- Adjustment cost models imply that inflation variability does not produce variation in output levels across firms, but does generate costs, uniform across firms. These don't seem to correspond to anything easily measurable.
- Mankiw-Reis inattention models imply no losses from anticipated inflation, but Calvo-like losses from unanticipated inflation.
- Rational inattention models, as we will see, imply costs of quite a different kind.

14. Is this a Phillips curve?

- Review: A.W. Phillips presented, in *Economica* in 1958, an empirical relation between unemployment and inflation.
- Why did this have such an impact?
  - The crude state of Keynesian theory for inflation at that time.
  - The increasing momentum behind quantitative econometric macro modeling.
  - Keynesian theory: "L-shaped supply". Phillips: Smooth curve.

15. What's new, and not so new, with the NK Phillips curve

- New: expectational term in the Phillips curve
- Not so new: "Real demand pressure" as the central element in inflation determination
- The not-so-new just reflects thinking of the expectational terms as minor qualifications, rather than central elements.
- Further discussion: Sims (2008)

16. The monetary policy to inflation causal chain

- old Keynes: Aggregate demand — monetary policy or G changes — directly impacts output and thereby unemployment.
- This then produces, probably with a delay, a change in inflation.
- New Keynesian: Expectations impact inflation directly, so the old dynamics are not necessarily there.

17. The causal chain, cont.

In class, I worked partially through a model with a NK Phillips curve and a constant real interest rate. Such a model can't produce even an approximate match to what we think we know about the effects of monetary policy, though. With a fixed real interest rate ρ, $r_t = \rho + E_t \pi_{t+1}$. Therefore a policy disturbance that raises r_t must make expected inflation next period increase. We usually think of tightening monetary policy as increasing r and then decreasing inflation with some delay, and this is obviously ruled out by the fixed-ρ assumption. If tighter monetary policy lowers inflation in a sustained way, it *lowers* r_t in such a model.

So it is essential to introduce the other component of the minimal NK model, the so-called "New Keynesian IS curve". It connects the real interest rate to the real activity level. With CRRA utility, it emerges as the consumption Euler equation

$$\frac{C_t^{-\gamma}}{P_t} = \beta R_t E_t \left[\frac{C_{t+1}^{-\gamma}}{P_{t+1}} \right] .$$

This would be the Fisher equation in the risk-neutral ($\gamma = 0$) case. For how monetary policy affects output and inflation with this equation in the model, see the exercise and the answer sheet for it.

References

Sims, C. (2008): "Inflation Expectations, Uncertainty, the Phillips Curve, and Monetary Policy," Discussion paper, Princeton University, http://sims.princeton.edu/yftp/CapeCod2008, Presented at the June, 2008 Boston Federal Reserve Bankmonetary policy conference.