CAPITAL TAXES

1. Why Capital Taxes Are Different

- Static analysis suggests that deadweight loss from taxation at rate τ is $0(\tau^2)$ — that is, that for small tax rates the ratio of deadweight loss to revenue is arbitrarily small.
- Consider a labor tax. Its effects on work, L, are $0(\tau)$. The equilibrium condition is $(1-\tau)f'(L) = -D_L U / D_C U$. But since the distortion involves trading C for leisure, and in the neighborhood of $\tau = 0$ the leisure gained is of the same utility as the consumption lost, the effects on welfare are small. There is of course a loss due to the C that gets appropriated for G, unless the government is optimizing so that G has utility that compensates for the decline in C. But the *additional* loss due to use of labor rather than lump-sum taxes is second-order.

2.

- Consider the steady-state effects of a tax on capital income. The equilibrium condition is $(1-\tau)D_k f(K, L) = \beta^{-1} + \delta - 1$. ($\delta$ is the depreciation rate.) In the standard growth model setup, $f(\ ,\ )$ is homogeneous of degree one, so $D_k f$ is a function of the capital-labor ratio $k = K/L$ alone. Furthermore, if the proceeds of the tax are rebated as lump sum transfers so that $C + I = f(K, L)$ and $\dot{K} = I - \delta K$, steady-state C/L is also a function of k alone: $C/L = f(k) - \delta k$. The tax decreases the steady-state capital-labor ratio and thereby also per capita consumption. The derivative of C/L with respect to τ is non-zero even at $\tau = 0$, so this is a first-order effect. In a model with inelastic labor supply, this directly implies a first-order effect on steady-state welfare. With elastic labor supply, the algebra becomes a little more complicated, but it remains true that there is a first-order negative effect on steady state welfare.
- But, a one-time, surprise capital levy is completely non-distorting.
- A temporary capital tax has the same $0(\tau^2)$ deadweight loss behavior as a labor tax.
- A permanent capital tax has the same $0(\tau^2)$ deadweight loss behavior as a labor tax, when this is measured in terms of discounted utility. The long run decline in utility from lower consumption in the future is, in the neighborhood of $\tau = 0$, exactly compensated for by the temporary rise in utility as dissaving allows temporarily higher consumption.

3. The nature of a optimal taxes

In a standard growth model with one capital good and one type of labor, suppose there is government debt and a single type of distorting tax — either on capital or on labor. And suppose the time path of this tax is chosen optimally, in a perfect foresight solution.

4. Capital tax τ

- Optimal τ is zero in the long run.
- It is as high as you like right now.
- This raises problems of time consistency.
- There is no steady state with fixed optimal $\tau \neq 0$.
- Optimality of socialism?
- In the answer to the capital tax exercise from the 2004 version of this course there is a detailed derivation of the result that the marginal welfare loss from a small constant capital tax is zero in the neighborhood of zero.

5. Labor tax ψ

- There is a steady state with $\tau \equiv 0$ and fixed $\psi \neq 0$ — one for each B.

6. Why the difference? The effects of "compounding".

A constant proportional capital tax changes the relative prices of current and future consumption. The effect is small in the current period, and over any finite number of future periods. But no matter how small τ is, $(1-\tau)^n$ eventually (for large enough n) is closer to zero than to one. So the distortion in the relative prices of present and future consumption is large for the distant future, even when τ is small. With discounted utility, this doesn't matter because the big distortions are also heavily discounted.

7. Seeing from FOC's that optimal capital tax is high at time zero

Suppose investment is in the hands of the consumer, who has a budget constraint of the form

$$\ldots + K_t + \ldots = \ldots + (1-\tau_t) r_t K_{t-1} + (1-\delta) K_{t-1} + \ldots$$

Here r is a current rental rate for machines and δ is a depreciation rate.

$$\partial K: \qquad \lambda_t = \beta E_t\big[\lambda_{t+1}\big(r_{t+1}(1-\tau_{t+1}) + 1 - \delta\big)\big] \qquad t = 0, \ldots, \infty$$

This private FOC is influenced by τ_t for $t \geq 1$, but not by τ_t for $t = 0$. No activity taking place at $t = 0$ or after is "taxed" by τ_0. Only the already-in-place capital K_{-1} is taxed.

8.

Of course if high $t = 0$ taxes finance high G_0, there are big real effects. But if the taxes simply finance retirement of debt and purchase of private assets, there are flows into private budget constraints that offset the flows out through taxes, so that overall there are no effects — except that the better initial net worth position of the government makes possible lower future taxes.

9. RICARDIAN EQUIVALENCE: THE REPRESENTATIVE AGENT

Consider a representative agent model in which the objective function is the expectation of some function U of current and future C and L and the constraint is

$$f(C_t + \tau_t + B_t - R_{t-1}B_{t-1}, K_t, K_{t-1}, L_t, A_t) = 0\,. \tag{1}$$

Though this may look unfamiliar, it includes as a special case, for example,

$$C_t + \tau_t + K_t - \delta K_{t-1} + B_t - R_{t-1}B_{t-1} = A_t F(K_{t-1}, L_t)\,. \tag{2}$$

Here τ represents lump sum taxes, B government debt, and R the gross interest rate on government debt. The agent chooses C, L, K, and B. The Euler equation FOC's are

$$\partial C: \qquad E_t D_{C_t} U = \lambda_t D_1 f_t \tag{3}$$

$$\partial L: \qquad E_t D_{L_t} U = \lambda_t D_4 f_t \tag{4}$$

$$\partial K: \qquad \lambda_t D_2 f_t = -\beta E_t[\lambda_{t+1} D_3 f_{t+1}] \tag{5}$$

$$\partial B: \qquad \lambda_t D_1 f_t = \beta R_t E_t[\lambda_{t+1} D_1 f_{t+1}]\,. \tag{6}$$

10. RICARDIAN EQUIVALENCE: GOVERNMENT

The government budget constraint is

$$B_t - R_{t-1}B_{t-1} + \tau_t = g_t\,. \tag{7}$$

We will treat the stochastic process for g as exogenously fixed and consider the effects of varying τ and B. In order to avoid allowing B to explode upward, which will generally be ruled out by the representative agent's transversality condition, τ will have to be set so it reacts to B. For example, if $\tau_t = -\phi_0 + \phi_1 B_{t-1}$, with ϕ_1 chosen so that $R_t - \phi_1 < 1$ for all t, then the government budget constraint (GBC) (7) becomes a stable difference equation in B.

Substituting the GBC into the private constraint gives us the social resource constraint (SRC):

$$f(C_t + g_t, K_t, K_{t-1}, L_t, A_t) = 0\,. \tag{8}$$

Note that the SRC, together with the three Euler equations (3-5), form a system of four equations in the four unknowns C, L, K, and λ. τ, B and R do not appear in these equations. In fact, these are the same four equations that would define the solution to a planner's problem in which the objective function was the same and the

SRC itself was the constraint. Thus we can conclude that any solution to the planner's problem is also an equilibrium for the economy with traded government debt, and that the equilibrium stochastic process for C, K, and L is invariant to the policy that sets the time path for τ and B, so long as the policy keeps B from exploding and is thus consistent with equilibrium.

11. What the result depends on, and doesn't depend on

- Doesn't require complete markets, in this single-agent model. Asset markets are competitive in the model and government debt is freely traded, but a complete menu of assets is not present.
- A single representative agent? If we had several agents with different tastes, but no uncertainty, the same result would hold. With uncertainty, the question is whether private agents can issue debt with the same characteristics as government debt. If so, the result holds. If not, the choice of tax policy can create variation in the risk characteristics of government debt and thereby affect the ability of agents to trade risk. With complete asset markets, we would be back to Ricardian equivalence even in this case.
- Requires no new births. As we'll see, new agents coming on the scene will undo Ricardian equivalence.
- Lump-sum taxes. We've assumed non-distorting taxes. If taxes are distorting, then their timing will matter.

12. An OLG model

A representative agent from the generation born at time t lives two periods, consuming $C_1(t)$ at time t in the first period of life and $C_2(t+1)$ at time $t+1$ in the second period of life. The constraints on the agent are

$$C_1(t) + S(t) \leq \bar{Y} \tag{9}$$

$$C_2(t+1) \leq \theta S(t) \tag{10}$$

$$S(t) \geq 0\,, \tag{11}$$

reflecting the fact that the agent is endowed with the single good in the first period of life and must save, earning real return θ on the saving, in order to consume in the second period.

The agent's problem is

$$\max_{C_1(t), C_2(t+1), S(t)} E_t U(C_1(t), C_2(t+1))\,. \tag{12}$$

The FOC's, assuming that the $S(t) \geq 0$ constraint is not binding, are

$$\partial C_1: \qquad E_t D_1 U(C_1(t), C_2(t+1)) = \lambda_t \tag{13}$$

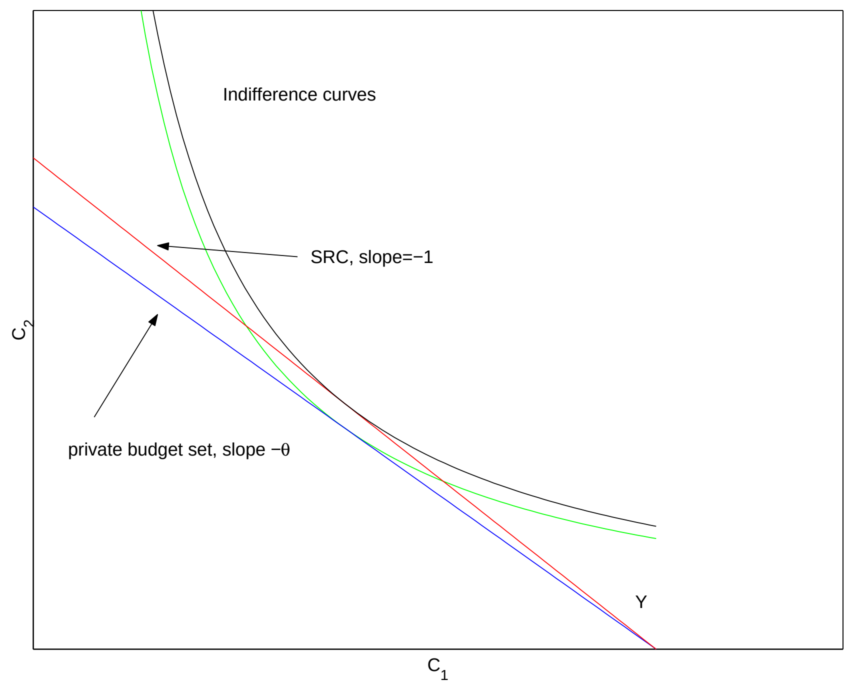
$$\partial C_2: \qquad D_2 U(C_1(t), C_2(t+1)) = \mu_{t+1} \tag{14}$$

$$\partial S: \qquad \lambda_t = \theta E_t \mu_{t+1}\,. \tag{15}$$

Eliminating the Lagrange multipliers, we arrive at the solved FOC

$$\frac{E_t D_1 U_{t+1}}{E_t D_2 U_{t+1}} = \theta\,. \tag{16}$$

For the issues we will be studying with this model for now, uncertainty is not central, so we will assume perfect foresight and drop the "E_t"'s, which allows us to use a familiar diagram to characterize equilibrium (labeled for the case $\theta < n = 1$):



This equilibrium involves no trade. Every generation provides for its own "retirement" by saving. This is called the autarchy solution. Under some conditions, a planner can do better. A planner would recognize that the social resource constraint at t is

$$N_t C_1(t) + N_{t-1} C_2(t) + N_t S_t \leq N_t \bar{Y} + \theta N_{t-1} S_{t-1} \tag{17}$$

$$S_t \geq 0\,. \tag{18}$$

For simplicity, assume $N_t = N_0 n^t$. If the planner chooses to make $C_1(t)$ and $C_2(t)$ constant over time and sets $S(t) \equiv 0$, then the planner's constraint becomes

$$C_1(t) + n^{-1} C_2(t) \leq \bar{Y}\,. \tag{19}$$

This constraint and the implied optimum is also shown in the figure. Clearly if $n > \theta$, the planner can achieve an equilibrium that improves on that obtained by individual saving. This situation, in which the return on private saving is below the population growth rate, is called **dynamic inefficiency**.

13. Government Debt and Taxation

With government debt, the private constraints become

$$C_1(t) + S_t + \frac{B_t}{P_t} + \tau_t \leq \bar{Y} \tag{20}$$

$$C_2(t+1) \leq \frac{R_t B_t}{P_{t+1}} + \theta S_t \tag{21}$$

$$B_t \geq 0\,, \qquad S_t \geq 0\,. \tag{22}$$

The government faces the budget constraint

$$N_t \frac{B_t}{P_t} + N_t \tau_t = N_{t-1} R_{t-1} \frac{B_{t-1}}{P_t}\,. \tag{23}$$

There is one new private FOC:

$$\partial B: \qquad \frac{\lambda_t}{P_t} = E_t \frac{\mu_{t+1} R_t}{P_{t+1}}\,, \tag{24}$$

assuming the $B_t \geq 0$ constraint is non-binding. With no uncertainty, when both B and S are non-zero, this FOC, together with the S FOC (15), implies

$$\theta = R_t \frac{P_t}{P_{t+1}}\,, \tag{25}$$

i.e. equal real rates of return on private storage and government debt.

14. Equilibrium with no taxes

With no taxes, the government simply rolls over the debt each period, setting

$$B_t N_t = B_{t-1} R_{t-1} N_{t-1}\,. \tag{26}$$

We consider whether there can be an equilibrium in which $C_1(t), C_2(t+1)$ are constant over time, no taxes are imposed, and there is no private storage. In such an equilibrium the amount of income saved in the form of bonds by the young would just match the amount of income consumed by the old each period. This, together with the constancy of C_1, C_2, implies

$$N_t \frac{B_t}{P_t} = N_{t-1} \frac{R_{t-1} B_{t-1}}{P_t} = N_{t-1} \frac{R_t B_t}{P_{t+1}}\,. \tag{27}$$

and therefore

$$\frac{R_t P_t}{P_{t+1}} = \frac{N_t}{N_{t-1}}\,, \tag{28}$$

i.e. the real rate of return on bonds is the rate of growth of population. To sustain the constancy of C_1 and C_2, the real rate of return must be constant, so such an equilibrium is possible only with a constant population growth rate. There are equilibria with non-constant population growth, but they do not have constant C_1 and C_2 across generations. If the real rate of return on bonds in this hypothetical equilibrium is less than θ, then this is not an equilibrium, as agents will see that by using the storage technology they could get a higher return. On the other hand, if the population growth rate exceeds θ, the equilibrium is sustainable and duplicates the planner's equilibrium we discussed above. Thus we have the conclusion that unbacked government debt (or "money", if we set $R = 1$) will sustain a competitive equilibrium that overcomes the dynamic inefficiency if autarchic equilibrium is inefficient, and will not disturb the autarchic equilibrium if it is efficient.

Note, however, that this result is fragile. There is always, even when $\theta < n$, besides the efficient equilibrium with valued debt, an inefficient one in which debt is valueless at all dates.

At the end of class Tuesday March 3, a student raised the question of whether there are equilibria, in the constant population growth case, in which $C_1(t), C_2(t+1)$ are not constant across t. The answer is yes. If at $t = 0$ the government offers nominal debt and people believe R_0P_0/P_1, the real return on this debt, is above θ but below $n = N_{t+1}/N_t$, all saving will be in the form of government debt, but in the following period, without taxes or transfers, the real value of the debt being redeemed is smaller than the previous generation's savings. If the real return on debt stayed at its previous value, the real value of the debt would fall short of the demand for saving. This would drive up the price and drive down the return on the bonds. Eventually (under some forms of U, immediately) the return on debt is driven down to θ, the real value of bonds shrinks monotonically, and the equilibrium approaches autarky.

In other words, there is a continuum of equilibria, in all but one of which the equilibrium converges to autarky and real savings $S > 0$ resumes.

15. Tax-backed debt, burden-shifting

Assumptions:

- Suppose for simplicity $\theta > n = 1$ and $\tau > 0$.
- The government must finance a real expense $g_0 > 0$, while $g_t = 0$ for $t > 0$.
- $\tau_t = 0$ except $\tau_T > 0$ for some single date T.

16. Reasoning

- In order for debt to be held by the public, it must pay the same return θ as the private storage technology, unless $S_t = 0$.

- At a date t when $\tau_t = 0$ and debt pays a real return θ, the young see government debt and private storage as equivalent assets, and since their endowment $\bar{Y}$ is unaffected by the availability of debt, their choice of $C_1(0)$ is unaffected by the presence of government debt in the economy.
- The government chooses a number of bonds B_0 and a price level P_0 such that $B_0/P_0 = g_0$, and it announces a nominal interest rate R_0.
- Since the debt must pay real return θ, the government must also convince the public that P_1 will emerge as $R_0 P_0/\theta$.

17. Evolution of the amount of debt

Let $\bar{C}_1$ be the first-period consumption choice of an agent with endowment $\bar{Y}$, rate of return θ, and no tax. So long as no tax is imposed, every generation in succession will set $C_1(t) = \bar{C}_1$. Furthermore, each such generation will choose the same level of second-period consumption, $\bar{C}_2$. The government budget constraint forces

$$\frac{B_t}{P_t} = \frac{R_{t-1}P_{t-1}}{P_t} \cdot \frac{B_{t-1}}{P_{t-1}} = \theta \frac{B_{t-1}}{P_{t-1}} . \tag{29}$$

Thus, since for all these generations the amount of savings, in the sense of endowment not consumed in the first period, is the same, S_t must shrink over time to allow real debt to grow, according to

$$S_t = \bar{Y} - \bar{C}_1 - \theta^t g_0 . \tag{30}$$

18. How it ends

Since S_t can't become negative, there is an upper bound on how long taxation can be postponed. If the debt is retired with a one-time tax in period T, the amount of the tax in that period is $\theta^T g_0$. Agents in the generation born at T therefore have their welfare reduced, because this tax shrinks their budget sets. The longer the tax is postponed, the greater the reduction in welfare for the generation that pays the tax. The generations before the taxed generation suffer no welfare consequences at all from the need to finance g_0.

The mechanism by which the tax burden is shifted is the reduction in real investment. Government debt "crowds out" private saving. So long as the taxation is postponed, the crowding out does not affect welfare. Only when the taxes are finally imposed does the reduced saving affect the welfare of a generation.

19. Discussion

In a model with a more realistic production technology, results are not quite this simple, but retain the same general character. The Diamond paper on the reading list works out a somewhat more realistic model.

Barro (1974) in a well-known paper showed that Ricardian equivalence reemerges if each agent has a "descendant" in the next generation, about whose welfare the agent cares. This argument is limited on the one side by the fact that not everyone in any actual population actually has caring intergenerational links. On the other side it is limited by the *reductio ad absurdum* in another well-known paper by Bernheim and Bagwell (1988), which shows that if agents must pair up to have descendants, which are then their joint descendants, then arguments like Barro's lead to the conclusion that not just public debt, but all of the taxes we usually think of as distorting, are in fact neutral.

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