CAPITAL TAX EXERCISE, DUE FRIDAY, 3/6

Consider a model in which a single representative agent solves

$$\max_{C,K,B} \sum_{t=0}^{\infty} \beta^t \big(\log C_t + \log(1 - L_t)\big) \qquad \text{subject to} \tag{1}$$

$$C_t + K_t + B_t = AK_{t-1} + L_t + R_{t-1}B_{t-1} - \tau L_t - \nu K_{t-1} \tag{2}$$

$$B \geq 0\,, \qquad K \geq 0\,. \tag{3}$$

Assume $A > 0$ and $\beta > 0$. At least at first, assume $A\beta > 1$, $\beta < 1$, which means that in the absence of taxation and government spending, the economy will grow steadily, though other cases are also of some interest.

The government sets τ, the labor tax, and ν, the capital tax. We assume there is no option of making these time-varying. The rates are being set at time $t = 0$ and must be kept constant thereafter. (This could be motivated, roughly speaking, by the idea that a commitment to a fixed tax rate will be believed by the public, but tax rates announced now to change in the future are not believed.) The government has a constant, exogenously fixed, burden of expenditures $\bar{g}$ to finance, so that the government budget constraint is

$$B_t + \tau L_t + \nu K_{t-1} = R_{t-1}B_{t-1} + \bar{g}\,. \tag{4}$$

Assuming the government wants to maximize representative agent welfare, find the optimal values of τ and ν. The answer will depend on initial B_{-1} and possibly also K_{-1}, which you should treat as given. Assume private agents and the government have perfect foresight, so there is no uncertainty. [You should be able to solve analytically for the time paths of C and L. You then can discount welfare analytically and get a formula for welfare in terms of τ and ν. You may need to use the fact that $\sum_0^\infty s a^s = a/(1-a)^2$.]