CAPITAL TAX EXERCISE, DUE FRIDAY, 3/6

Consider a model in which a single representative agent solves

$$\max_{C,K,B} \sum_{t=0}^{\infty} \beta^t \left( \log C_t + \log (1 - L_t) \right) \quad \text{subject to} \quad (1)$$

$$C_t + K_t + B_t = AK_{t-1} + L_t + R_{t-1}B_{t-1} - \tau L_t - vK_{t-1} \quad (2)$$

$$B \geq 0, \quad K \geq 0. \quad (3)$$

Assume $A > 0$ and $\beta > 0$. At least at first, assume $A\beta > 1$, $\beta < 1$, which means that in the absence of taxation and government spending, the economy will grow steadily, though other cases are also of some interest.

The government sets $\tau$, the labor tax, and $v$, the capital tax. We assume there is no option of making these time-varying. The rates are being set at time $t = 0$ and must be kept constant thereafter. (This could be motivated, roughly speaking, by the idea that a commitment to a fixed tax rate will be believed by the public, but tax rates announced now to change in the future are not believed.) The government has a constant, exogenously fixed, burden of expenditures $\bar{g}$ to finance, so that the government budget constraint is

$$B_t + \tau L_t + vK_{t-1} = R_{t-1}B_{t-1} + \bar{g}. \quad (4)$$

Assuming the government wants to maximize representative agent welfare, find the optimal values of $\tau$ and $v$. The answer will depend on initial $B_{-1}$ and possibly also $K_{-1}$, which you should treat as given. Assume private agents and the government have perfect foresight, so there is no uncertainty. [You should be able to solve analytically for the time paths of $C$ and $L$. You then can discount welfare analytically and get a formula for welfare in terms of $\tau$ and $v$. You may need to use the fact that $\sum_{0}^{\infty} sa^s = a / (1 - a)^2$.]