ANSWERS TO 2/19 EXERCISE

The equations of the model are

$$\text{SRC}: \quad C_t = Y_t \tag{1}$$

$$\text{GBC}: \quad b_t + \tau_t = R_{t-1}\frac{b_{t-1}}{\pi_t} \tag{2}$$

$$\text{FOC}: \quad 1 = \beta R_t E_t\left[\frac{P_t C_t}{P_{t+1}C_{t+1}}\right] \tag{3}$$

$$\text{Monetary Policy}: \quad R_t = \bar{R}\varepsilon_{Mt} \tag{4}$$

$$\text{Fiscal Policy}: \quad \tau_t = \bar{\tau} + \varepsilon_{Ft}\,. \tag{5}$$

Linearize the whole five-equation system around its steady state and use gensys or some other RE solver to compute the solution, checking whether existence and uniqueness hold. Use $\beta = .9$, $\bar{R} = \beta^{-1}$, $\bar{\tau} = 1$. Compare the impulse responses of π_t to ε_M, ε_F and Y_t shocks to what was found above for the two-equation system. What would happen in this system with policy given by (4-5) if we tried the previous strategy of using the monetary policy equation and the FOC to form a two-equation system and tried to solve it in isolation?

The steady state is $\bar{C} = \bar{Y}$, $\bar{R} = \beta^{-1}$, $\bar{b} = \bar{\tau}/(\beta^{-1} - 1)$, $\bar{\pi} = 1$.

With these policies, The system can easily be reduced to a pair of equations, which I hadn't realized. Letting $g_t = Y_t/Y_{t-1}$ and $\pi_t = P_t/P_{t-1}$, we can write the FOC and the GBC as

$$1 = \varepsilon_{Mt}E_t[(\pi_{t+1}g_{t+1})^{-1}]$$
$$b_t = \beta^{-1}b_{t-1}\pi_t^{-1} - \bar{\tau} - \varepsilon_{Ft}\,.$$

This is a two-equation system in the two variables b and π, driven by g_t, ε_{Mt} and ε_{Ft} as exogenous shocks. It is possible to solve this system analytically, but for this exercise we are linearizing. Linearizing w.r.t. $\log\varepsilon_M$, $\log\pi$, $\log g$, b and ε_F, we arrive at

$$\tilde{\pi}_{t+1} = \tilde{\varepsilon}_M t - \tilde{g}_{t+1} + \eta_{t+1}$$
$$\tilde{b}_t = \beta^{-1}\tilde{b}_{t-1} - \beta^{-1}\bar{b}\tilde{\pi}_t - \varepsilon_{Ft}\,,$$

where the ˜'s denote deviations from steady states.

Note that ε_{Mt} enters the first equation lagged once relative to the π_{t+1} variable. This means that if we just lump it in with the exogenous shocks, we will have a predictable exogenous shock. Also, while it would be convenient here to assume $\tilde{g}_t$ i.i.d. with mean zero, this would make our impulse responses non-comparable to those of the stable solution to the system in the notes with active money and passive fiscal policy. To make the output of gensys more directly usable in computing impulse responses, it is handy to convert the system to one in which shocks all appear without lags, by defining new state variables $z_t = \log \varepsilon_{Mt}$ and $w_t = \log Y_t$. Then rewriting in matrix notation (and dropping the ˜'s) we get

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ \beta^{-1}\bar{b} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ b_t \\ z_t \\ w_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & \beta^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ b_{t-1} \\ z_{t-1} \\ w_{t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{M,t} \\ \varepsilon_{Ft} \\ \tilde{Y}_t \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \eta_t \,,$$

where as usual we are using η_t to designate a one-step-ahead prediction error about which we know only $E_t \eta_{t+1} = 0$. This model has a simple enough structure that we could easily solve this linearized version directly by deriving an unstable equation in a single variable and imposing stability. But `gensys` is also easy, and it will make impulse responses a bit easier.

`gensys` confirms that we have existence and uniqueness of a stable solution, and gives us as `G1`

	pi	b	z	w
pi	0.00	0.11	-0.00	-0.00
b	0.00	-0.00	0.00	0.00
z	0.00	-0.00	0.00	0.00
w	0.00	-0.00	0.00	-0.00

(The (1,2) element of `G1` is actually 1/9). The impulse responses can be found by converting $G1$ to a $3 \times 3 \times 1$ array and giving it as an argument to `impulsdtrf`, with the `impact` matrix as `smat`. However, since in this case the `G1` matrix is rank 0, with its square equal to zero, one might as well just note that the first element of the sequence of impulse response matrices is the `impact` matrix itself, and the second is `G1 %*% impact`. The first and second impulse response matrices are then

	eps_M	eps_F	y
pi	-0.90	-0.10	-0.90
b	9.00	0.00	9.00
z	1.00	0.00	-0.00
w	0.00	0.00	1.00

	eps_M	eps_F	y
pi	1.00	0.00	1.00
b	0.00	0.00	0.00
z	0.00	0.00	0.00
w	0.00	-0.00	-0.00

Note that a surprise monetary contraction that increases the interest rate by one percentage point reduces current inflation by 0.9% in the current period, but that inflation then springs back up by a full percentage point in the following period. This is a symptom of the fact that with this active-fiscal policy configuration monetary contraction can have at best a temporary restraining effect on inflation. If the monetary authority raised interest rates in response to an increase in inflation, and then raised them again in response to the rebound inflation it had itself caused, and repeated this behavior for several periods, the price level would be higher than when the process started, and higher by more the longer they had persisted in this behavior.

Note also that because ε_M is measured in log units, while ε_F is a shock to the level of τ, the the relative sizes of the responses to ε_F and ε_M have to be interpreted with care. A change of .01 in ε_M in the linearized system is a one percentage point change in the interest rate, while a similar change in ε_F is a one per cent change in τ (at our assumed $\bar{\tau}$ of 1). The change of .01 in the interest rate at the steady-state interest rate of 11.1% is about a 10% change in the level of the interest rate. A 10% change in the level of τ via ε_F has about the same impact as such a change in the interest rate.

In the previous system, with active money and passive fiscal policy, the impulse responses were trivial, with only contemporaneous impacts of shocks. The inflation rate responded negatively to ε_M and y, but not at all to ε_F. In this system, the contemporaneous response to ε_M and Y is similar, but now ε_F also affects inflation. In the former system (though we did not in the notes bring this out) ε_F does have an impact on b. In this system only ε_M and Y have any impact on b. And of course in this system the negative impact effects of ε_M and Y on π are more than reversed by the lagged effects.

If we had tried to solve this system by looking at the FOC and the monetary policy rule alone, we would have concluded that they imply that for any $\kappa > 0$, $P_t Y_t = \kappa / \varepsilon_{Mt}$ is a solution, so long as $E[\varepsilon_{Mt}] = 1$ as we assumed. Since κ is indeterminate, the price level is at each t indeterminate. It is only by bringing in the government budget constraint and fiscal policy that we see that the price level is determinate.